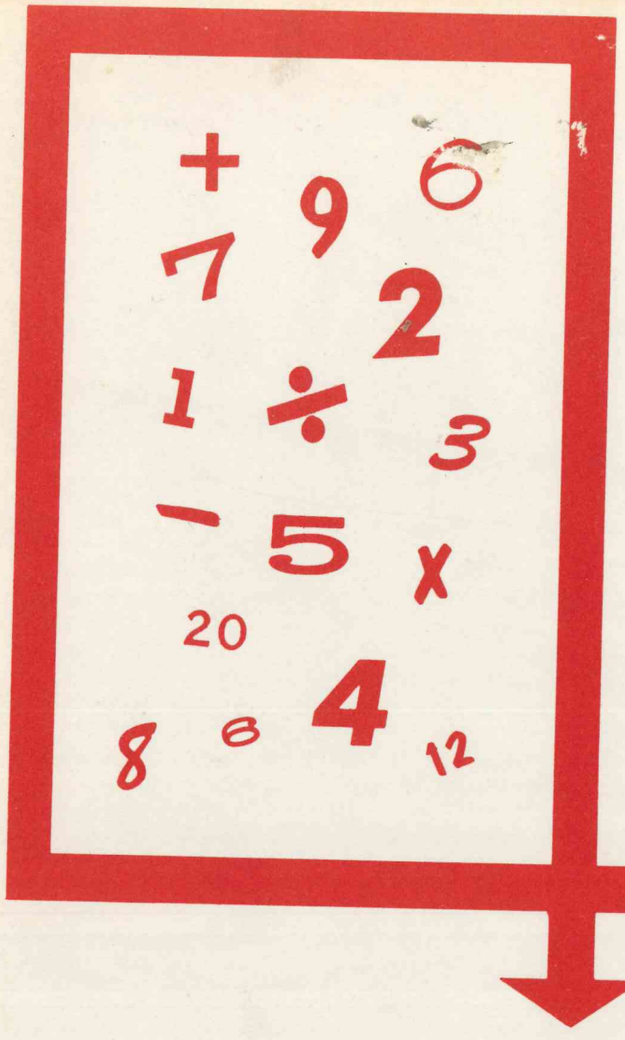

IGS

Practical Arithmetic

Whole Numbers

By

W. M. SPANGLER, M. A.

6750A
Part 1
Edition 1

YOUR AUTHOR

This text, "Practical Arithmetic," in six parts, has been specially written by William M. Spangler, Director of the School of Mathematics, in which the instructional service for this text is provided. In preparing "Practical Arithmetic," Mr. Spangler's aim has been to ensure that the material presented to the student meets the high standards of technical accuracy, ready application, completeness, and readability to which every ICS text must conform.

Mr. Spangler is a graduate of Bucknell University. He is a member of the National Council of Teachers of Mathematics and has had extensive experience in teaching and writing.

Practical Arithmetic

PART 1

Whole Numbers

By

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Serial 6750A

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International Correspondence Schools, Scranton, Pennsylvania
International Correspondence Schools, Canadian Ltd., Montreal, Canada

What This Text Covers . . .

This text deals with the four basic arithmetic operations as they apply to whole numbers. Your progress in the study of whole numbers depends on you. The time that you spend on it and the effort you put into it will determine how soon and how well you master it. This breakdown of the text into sections should help you in your study. Completing each section is a step forward. Do not try to study any new section until you have mastered all the practice problems and programmed instruction in the preceding section.

1. WHAT ARE WHOLE NUMBERS? ..... Pages 1 to 25
   In this section you'll find out what whole numbers are. And you'll learn how to read and write them. As you study, you'll meet some new words. Be sure that you know what they mean and how to spell them.

2. ADDITION ..... Pages 26 to 40
   Here you'll find out how to prepare figures for adding, and you'll learn the easy way to add columns of figures.

3. SUBTRACTION ..... Pages 41 to 49
   This is an easy but important section. Here again you'll learn some new words and their meanings.

4. MULTIPLICATION ..... Pages 50 to 55
   You'll find out that multiplication is simply an easy way to do repeated additions. Multiplication tables are included here to help you. Memorize them carefully.

5. DIVISION ..... Pages 56 to 67
   Here you'll learn how to divide one number by another. In this operation you'll use both subtraction and multiplication.

6750A

Practical Arithmetic

PART 1

Whole Numbers

Getting Acquainted with Whole Numbers

What is Arithmetic?

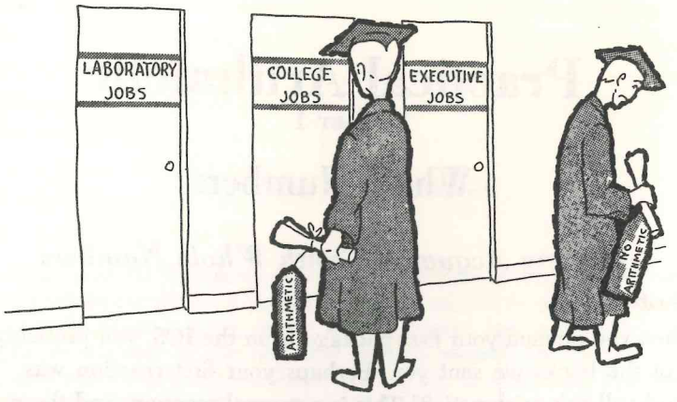
1. When you opened your first package from the ICS, you probably looked at the titles of the books we sent you. Perhaps your first reaction was, "Why do I have to study all this arithmetic?" This is a normal reaction, and the question is a good one. In the next few pages, we are going to point out reasons for studying arithmetic. And as you go on with your study, you'll discover many other reasons for yourself. But before we show you how important arithmetic is, let's find out *what* arithmetic is.

The dictionary definition of arithmetic goes something like this: "the art of computation by the use of positive, real numbers." You might not fully understand this definition, but don't let that bother you. For now, let's just say that arithmetic is the study of numbers and how they are used. This point of view naturally divides our subject into two parts: learning what numbers are and learning what we can do with them. This is the basic plan we will follow in teaching you arithmetic.

Maybe our simplified definition of arithmetic has done nothing to overcome any fears you might have about the subject. To hear some people talk, you'd think that arithmetic was something to be avoided, something to be left to mathematicians. Such people have developed the wrong attitude toward the subject. Perhaps you yourself have had some unpleasant experiences with arithmetic which left you with mental "roadblocks." It's only natural then for you to wince a bit every time something reminds you of the subject. Now we aren't going to start off by saying that arithmetic is easy, because that isn't quite true. Nothing is easy until you learn how to do it. There are many things that you can learn only by doing. For example, you can't learn to swim by watching other people swim. You must get into the water yourself. Neither can you learn arithmetic just by watching.

Why Should You Study Arithmetic?

2. Now we're ready to take up the question, Why study all this arithmetic? Let's look at it this way. In your grandfather's day a person could get along well enough even if he couldn't read. That isn't true today. Even if you read only the sports page or the comics, try to imagine what it would be like if you couldn't



read at all. Well, it's the same way with arithmetic. In your grandfather's day, a person could get along pretty well if he could do only a little figuring. But that is no longer true. Whatever you do for a living, you still need to know how much money you'll get on payday. In fact, unless you take a lot for granted, you probably figure out in advance what your paycheck will be. When you do this, you are using arithmetic, whether you realize it or not.

Ask your mother or your wife if she uses arithmetic. If she says, "No, I've forgotten all about it," or something to that effect, ask her what she does when she orders groceries, follows a recipe, or prepares her household budget. She uses arithmetic, of course! And when she decides that she can't afford a new hat or a new pair of shoes for Junior, she is using arithmetic too.

The whole point here is that we all use arithmetic. But now you might say, "OK, I'll admit arithmetic is necessary, but why so much of it? It's the other parts of the course I'm really interested in." We can understand this reaction too, but even though arithmetic was part of your earlier education, you have probably been using regularly only a few of the principles you learned. You are like a carpenter who must get some new tools and sharpen his old ones in order to make progress in his work. You need a sound knowledge of arithmetic to make progress in your course. Moreover, arithmetic is essential in any field of business or industry, from accounting to X-ray technology.

At first glance, the arithmetic you must do may look like a pretty big dose. You can be sure, however, that we've given you only as much arithmetic as you'll actually need. We have presented this arithmetic in easy stages, and you'll find it much simpler than you expect. So there's no need to be dismayed by the task before you. Remember that a weak background in mathematics has held back many people who want to get ahead. It's the major block to college admissions, to progress in technical schools, and to success in business and industry.

Getting into the Proper Frame of Mind

3. In the study of arithmetic, as with any endeavor, your success will depend largely upon your adopting the proper frame of mind. The first step in this direction is to drop any notion that mathematics in general is something that only "brains" can understand. If you're inclined to think that way, you're going to be pleasantly surprised, because soon you'll have a clear understanding of arithmetic, one of the simplest branches of mathematics. Don't put any obstacles in your path by starting off with a disinterested attitude. Study your instruction texts properly, and you'll find that arithmetic can be fascinating. As you master each operation, you'll become more and more enthusiastic about your studies.

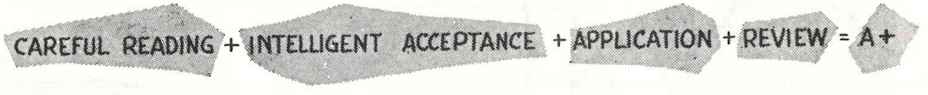
By now, we hope you are sold on the idea of studying arithmetic. So let's end the sales talk and get down to the business of how to study arithmetic.

How to Study Arithmetic

4. The suggestions given here will help you avoid poor study habits, which can lead only to discouragement and putting things off. *Please don't dismiss these hints as unimportant.* You'll find them helpful, not only in arithmetic but in other subjects as well.

1) *Cultivating good study habits.* First, find a place where you won't be bothered by television, radio, or any other distractions.

Learning is the process of connecting a new idea with something you've already learned. So, before you tackle something new, spend just a few minutes mentally gathering together all the things you already know about the subject. Then you're ready to go on to the next idea. Later on, you may find it necessary to go back and review some earlier topic in order to get the most out of a new topic. Do this by all means. After you have a good idea of a particular operation, you can go on to the next topic.



2) *Studying the new material.* There are many rules for proper study. You've probably heard the admonition, "Pay attention to the teacher." That's good advice for the classroom and it's good here too. We are your teachers, and this text is our only way of "talking" to you. Therefore, you must read very carefully what we have to say and you must understand it. Actually, you have a big advantage over classroom students. You can go back to any statement and read it over and over again until it sinks in. There will be many times when you will have to do this very thing. Be sure you know exactly what each statement means. If you do not fully understand a word, look it up in a good dictionary. Once you understand what we have stated, your next step is to accept it. Don't doubt the statement or

Rules of the road . . .

You're going too fast if . . . . . .

You start a new section without reviewing what you studied yesterday.

You break the rules to "save time."

You don't take an extra minute or two to check your work.

Have a definite time and place to study.

Concentrate! Don't let *anything* distract you.

Allow plenty of time to do your work well.

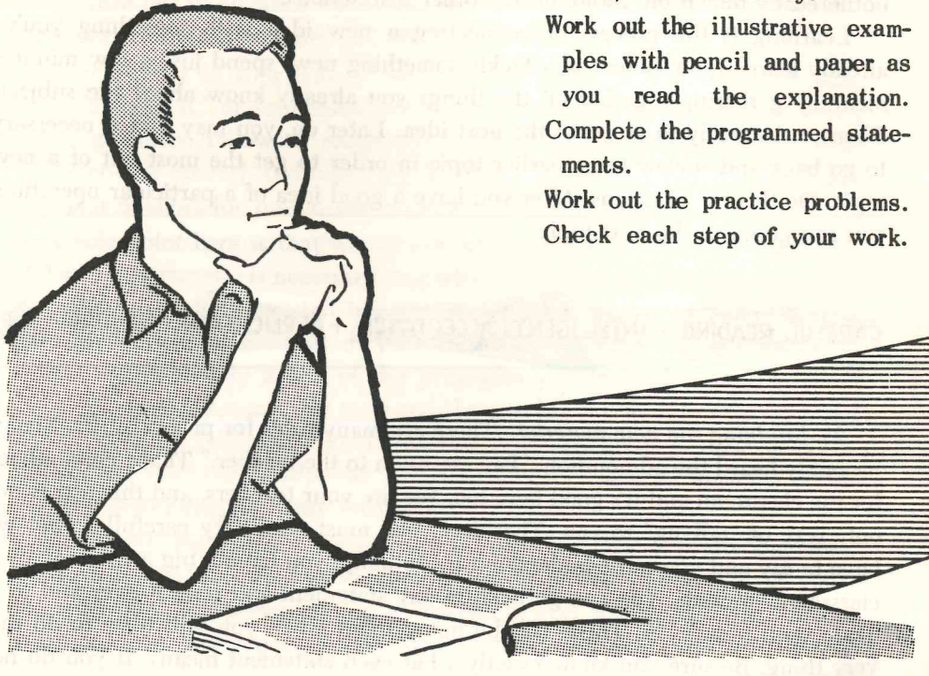
Read the subject matter carefully; if you don't understand a word, use a dictionary.

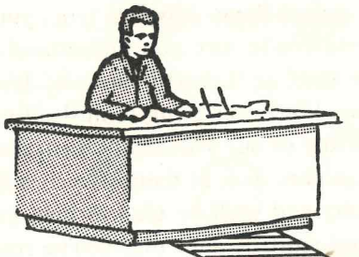
Work out the illustrative examples with pencil and paper as you read the explanation.

Complete the programmed statements.

Work out the practice problems.

Check each step of your work.





When the problems become difficult. .

Don't give up!

Keep trying even if you have to go slowly.

Read the instructions again. Make sure you understand them.

Make up an easy problem similar to the difficult one. Do it; then solve the hard problem, using the same procedure.

When you postpone your study time till "later on."

When you don't follow instructions. "Shortcuts" can be disastrous.

When you stop studying an operation or procedure before you have mastered it.

When you don't ask us for help on the things you don't understand.

When tempted to slide over the programmed instructions.

When tempted to skip a practice problem or two.

question its truth. This doesn't mean, of course, that you should stop thinking for yourself and accept things blindly. True, you should mull each thought over in your mind until you're sure you understand it. But never just dismiss it, saying "That can't be true" or "I don't believe it." It's up to you to convince yourself that it must be true. We aren't going to mislead you by making false or contradictory statements. If any of our statements seem contradictory or just don't make sense to you, take another look at them. If you still have doubts, tell us where you're having difficulty and we'll be glad to help you understand it better.

Finally, you must be sure that you've really learned what you have studied. It's very easy to say, "Oh, that's easy," or "Sure, I can do that." You'll be surprised to find how hard it sometimes is to do these "easy" things by yourself. You'll meet many problems that we have solved for you. They are called illustrative examples. Of course, don't just read the steps of a problem; work through them just as we have done. Then close the book and see whether you can solve the problem without looking at our work. It's a good plan to try some of these solved problems again, say several nights later. Cover up our solutions and see if you can still solve the problems. If you get stuck, take another glance at our work. In this way, you'll learn how to figure out solutions for yourself. Now you're ready for the next step.

3) *Applying your new knowledge.* In each text you'll find groups of problems or questions called practice problems. You're to solve them yourself, and to check your answers against the ones in the back of the text. Many of these problems will be very similar to the illustrative examples we have solved for you. Therefore, solving the practice problems will give you the opportunity to apply what you have learned. Think about each problem, reason it through, and then solve it. Don't just take the numbers given in the problem and try to juggle them around to get the answer. The primary object is not to get our answer but to solve the problem by using a correct method. The correct answer is the natural result. This is one place where you can cheat yourself. But if you do, you will only hurt yourself. Be honest. Solve each problem, and make sure you know how to do it correctly.

In arithmetic, skipping over a principle today means tripping tomorrow. So if there is some problem that you can't solve, don't skip it. Instead, go back and restudy the preceding articles of the text. If you're a bit weak on one of the problems we solved for you, study it again until you understand it thoroughly. Then go back to the problem you couldn't do and try again. If you still can't get it, write to us for help. Send us the work you've done on the problem so that we can see where you were making your mistake. Be sure to send your work, because we'll be better able to give you some good suggestions if we see just what you've done.

The solving of practice problems is a most important step. It is only by applying what you have studied that you will find out how much you know. This practice work will also fix the methods and ideas you've studied more firmly in

your mind so that you won't forget them. You may think that doing all these problems is hampering your progress, but it isn't. Actually, the more thorough you are at this stage, the better the understanding you will have. In other words, the harder you drive yourself in this work, the better the working knowledge you will have. This will save time later on.

4) *Testing yourself.* The final step is testing yourself to see just how well you are doing. Don't be easy with yourself. Don't give yourself a pat on the back until you deserve it. If you can't get the correct answer to a problem, keep at that problem until you do. Then mark the problem so that you can try it again at another time. A little later on, go back to any problems that you have marked and do each one of them again. If you get them all right, that's fine. But if you go wrong on any one of them, you have probably missed some important point. If, after you have tested yourself, you find that you are not as good as you should be, it's up to you to study and practice until your tests show that you are up to standard. Don't be satisfied by anything less than a correct answer for every problem.

Check Your Learning

One of the newest concepts in educational methods has been adapted for use in the *Practical Arithmetic* series. This is known as programmed instruction, or Check Your

Learning, and represents a milestone in teaching and learning methods. You've probably already heard a good deal about this new concept; now here is your chance to put it to work for yourself. We at ICS are proud to make it available to you.

Used properly, the programmed instruction we've included in these texts will speed your learning process and impress the most important points of information more firmly on your mind. By means of this programmed instruction, you are exposed a second time to the essential facts and the important procedures explained in the text material. All this is accomplished without the painful process of "hunting and finding" to pinpoint the most important facts. You'll also find that the programmed instruction provides an easy means of self-testing on important material.

There are groups of programmed statements, numbered consecutively, scattered throughout this text. Most of these statements require you to supply a one- or two-word answer. In others, a choice of two words is given and you must select the word you think is correct. You'll find the correct answer slightly below and to the right of each statement. Do not look at our answer until you've tested your understanding of a statement by completing it yourself. Cover the answer with a piece of paper and write your own answer in the blank provided. Then compare your answer with the one given at the right. If your answer is correct, go on to the next statement. If your answer is wrong, review the text material on which the statement is based until you understand your error.

1. Arithmetic is the study of _____ and how they are used.

(numbers)

2. The process of connecting a new idea with something you already know is called _____.

(learning)

3. When you don't understand the meaning of a word, you should look it up in a _____.

(dictionary)

4. In our textbooks, there will be many examples solved for you. They are called _____ examples.

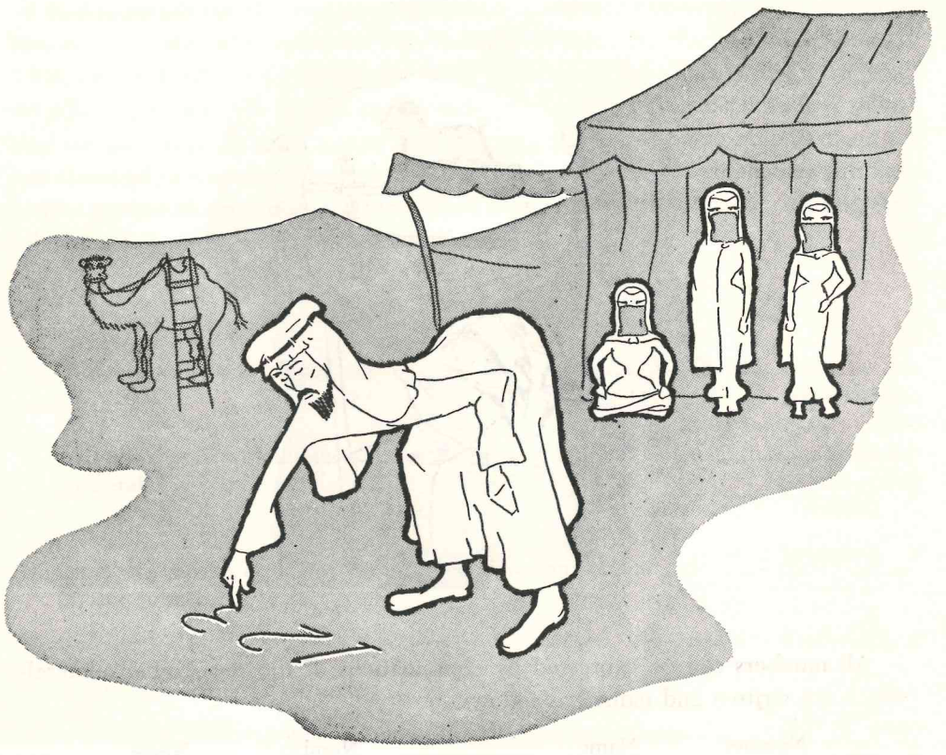
(illustrative)

5. As you study this text, solve all of the _____ problems and compare your answers with those in the back of the text.

(practice)

6. The object of the practice problems is to see whether you can solve them by using a correct _____.

(method)

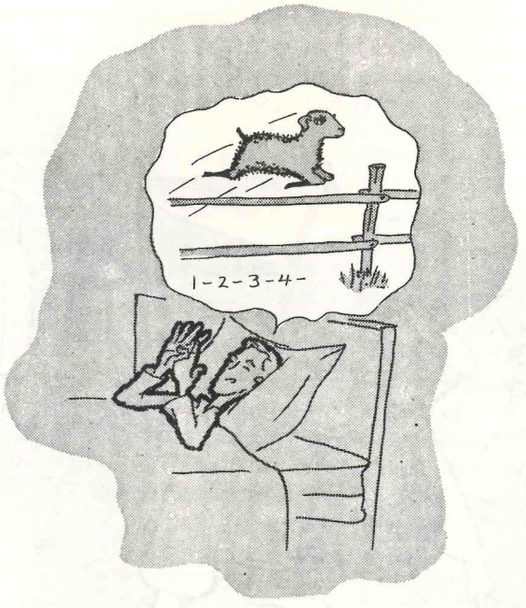


Our Number System

What Is a Number?

5. Right at the beginning of this text we told you that the study of arithmetic can be broken down into two parts: 1) learning what numbers are and 2) learning what we can do with numbers. Therefore, the first question that must be answered is, What is a number? The answer is: *Any word or symbol that tells us How many? or How much? is a number.* Thus if you say, "I have two eyes," "two" is a number because it tells how many eyes you have. When we say that there are five men in the room, "five" is a number because it answers the question, How many men are there?

Numbers can be expressed by words or by symbols. For convenience in working with numbers, we write them as symbols called Arabic numerals. These symbols were introduced by the Arabs long ago, which accounts for the name. The Arabic number system is an old system but a good one.



All numbers can be expressed by combinations of the ten Arabic numerals, which are written and named as follows:

Number	Name	Number	Name
0	zero	5	five
1	one	6	six
2	two	7	seven
3	three	8	eight
4	four	9	nine

These individual numerals are called figures, but when two or more figures are combined to form a larger number, they are called digits. The figure 0 is sometimes called ought or cipher instead of zero. The figure 1 is called one, or unity. The name "unity" comes from the fact that 1 represents a single thing, which we call a unit. We have listed these figures as they increase in value. Each figure is just one greater than the one before. In other words, 1 is one greater than zero, 2 is one greater than 1, 5 is one greater than 4, and 9 is one greater than 8. Because these numbers represent whole things, they are called *whole numbers.* Whole numbers are also called *integers.* You'll get a clearer picture of what a whole number is if you think of the number of men in a room, the number of cars in a parking lot, or the number of rivets in a steel plate. Each of these is a certain quantity of whole things. In this text you'll learn more about whole numbers and what can be done with them.

Counting

6. When we say the names of the whole numbers, beginning with one and continuing in the order in which they increase in value by one, we are counting. Thus we count to five by saying one, two, three, four, five. This is just what you do when you count the fingers and thumb on one of your hands. The first thing that we learn in arithmetic is counting! You may think that being able to count is not absolutely necessary. But it is, for it is only by learning to count that you get a clear picture of our number system. Even if you know how to count, reading the following few pages will be worthwhile.

Check Your Learning

7. Numbers can be expressed either by words or by _____. (symbols)

8. When we work with numbers, we write them as symbols called _____ numerals. (Arabic)

9. Numbers can be expressed by using combinations of the _____ Arabic numerals. (ten)

10. Individual numerals are called _____. (figures)

11. We sometimes read 0 as ought or _____ instead of zero. (cipher)

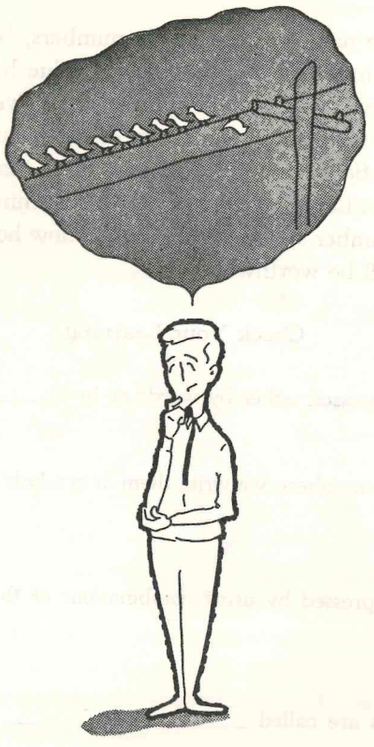
12. The figure 1 is called one, or _____. (unity)

13. Since numbers such as 1, 5, 7, and 9 represent whole things, they are called _____ numbers. (whole)

14. Since 5 is 1 greater than 4, we say that 5 has a greater _____ than 4. (value)

15. Whole numbers are also called _____. (integers)

16. When we say the names of whole numbers consecutively, or as they increase in value – say one, two, three, four, five – we are _____. (counting)



Numbers Having Two Digits

7. If you wanted to count the fingers and thumbs on both your hands, you would say one, two, three, and so forth, until you came to nine. But then you would discover that there was one finger still to be counted. What should you call it, and what figure would be used to represent it? The number which is one greater than 9 is called ten. In it written 10. The next number is eleven (written 11), the next twelve (12), and so forth, as the following list shows.

Number	Name	Number	Name
10	ten	15	fifteen
11	eleven	16	sixteen
12	twelve	17	seventeen
13	thirteen	18	eighteen
14	fourteen	19	nineteen

Notice that this new series of ten numbers has been formed by putting the number 1 in front of each of the numerals in the first series, given in Art. 5. Observe that the names of the last seven numbers in the new series end in *teen*. In fact, the names of four of these numbers consist of the name of the last digit in each number followed by *teen*. These numbers are 14, 16, 17, and 19. The

names of the other three, namely 13, 15, and 18, follow the same pattern, except for changes in spelling. That is, instead of threeteen we have thirteen, instead of fiveteen we have fifteen, and instead of eightteen we have eighteen.

When we get to 19, we are faced with the same problem that we met when we reached 9. Remember that we formed the second series of ten numbers by putting the number 1 in front of the first series. Isn't it reasonable then that the third series of ten numbers should be formed by putting the number 2 in front of the first series? Sure it is, and that is just what we do. These numbers and their names are as follows:

Number	Name	Number	Name
20	twenty	25	twenty-five
21	twenty-one	26	twenty-six
22	twenty-two	27	twenty-seven
23	twenty-three	28	twenty-eight
24	twenty-four	29	twenty-nine

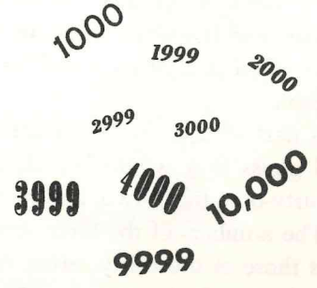
As you look at this series, you'll notice that the name of the first number is a new word, "twenty." The names of the other numbers are formed by putting the name of the last digit of each number after the word "twenty." The two words are always separated by a hyphen.

The number that comes next after 29 is 30. Its name is thirty. The next nine numbers are, just as you'd guess they would be, 31, 32, 33, 34, 35, 36, 37, 38, and 39. Their names are thirty-one, thirty-two, and so forth, to thirty-nine. After 39 comes 40, called forty. The numbers of the forty series and their names follow exactly the same pattern as those of the thirty series. After the forty series come the fifty, sixty, seventy, eighty, and ninety series. The first number of each of these series is written in numerals like this: 50, 60, 70, 80, and 90. And the next nine numbers of each of these series are formed and named exactly as for the other series.

Numbers Having Three Digits

8. When we get to 99, we've gone just as far as we can with numbers of two digits. Here we're faced with a problem just like the one we met when we got to 9. After 9, the last of the single-digit series, we started the two-digit series. Now at the end of the two-digit series, we start the three-digit series. Before we show you what this three-digit series is — before we even discuss it — we want you to think about the two-digit series for a moment. Ask yourself what you did when you formed the number after 9, the one after 19, 29, 39, and so on to 89. Didn't you increase the digit preceding 9 by 1 and change the 9 to 0? For example, after 9 came 10, after 19 came 20, and so forth. Notice that in each of these cases the 9 has become 0 and the digit preceding the 9 has been increased by one. This is the key to our number system. It is this fact that takes away any mystery there might be about how numbers are formed.

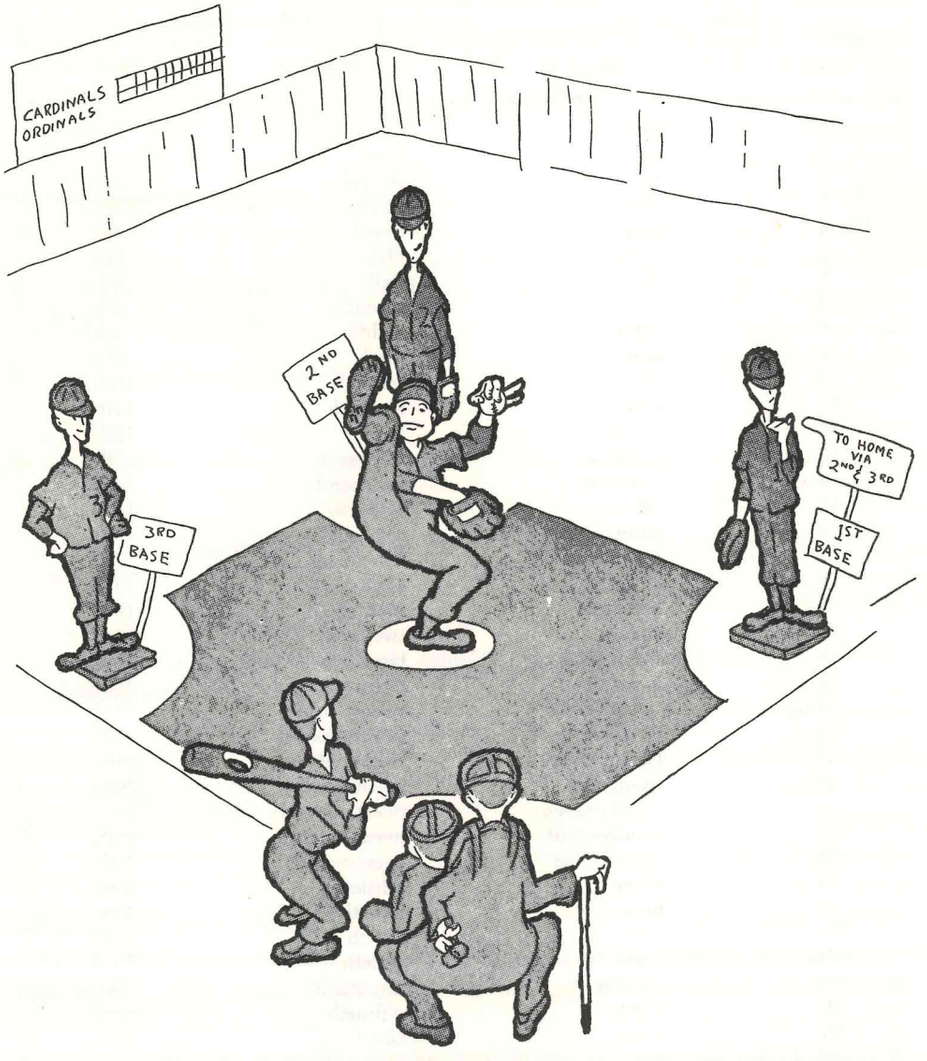
Now let's apply this simple fact to 99. First, increase the left-hand 9 by 1. This gives you 10, doesn't it? Now change the other 9 to a 0, and what do you get? You should have a number that looks like this: 100. This number is called one hundred and is the first number of the three-digit series. The next number is 101, called one hundred one. The next is 102, called one hundred two. In fact, we go through the entire series of single-digit numbers in this way and through all the two-digit numbers as well until we come to 199. The name of any of these numbers is the same as we learned before, except that we always say one hundred before it. When we get to 199, we again change the final 9 to a zero and increase th digits (19) in front of it by 1. This gives us 200, which is called two hundred. From here we begin again with 1 and count to 99, saying two hundred before each number. Thus we say two hundred one, two hundred two, and so on, up to two hundred ninety-nine. In like manner, we continue to 300, 400, 500, 600, 700, 800, 900, until we come to 999.



Numbers Having Four or More Digits

9. What is the largest three-digit number? Think about this question and associate it with what you learned in the two-digit series. It shouldn't take you very long to conclude that the largest three-digit number is 999. Now where do we go from here? Recall what we did before when we had a number ending with a 9. We changed the last 9 to 0 and increased the digits before it by 1. Well, what number is one greater than 99 (the two digits before the last 9 in 999)? It's 100, isn't it? And when you put a zero on the end of 100, you get 1000. The name of this four-digit number is one thousand. The next number is 1001 and the next 1002. These are read one thousand one and one thousand two. Here we again repeat all the numbers we have already learned, putting the words "one thousand" before each. Eventually, we'll come to 1999. Can you explain why the next number is 2000? Remember the rule for a number that ends with a 9? Two thousand is the name for 2000. From 2000 we go up to 2999 and then to 3000 (three thousand). From 3000 we go up to 3999 and then to 4000 (four thousand). Continue this until you come to 9999, which is read nine thousand nine hundred ninety-nine.

After 9999 comes 10000, which is read ten thousand. In fact, you can keep on writing numbers in this manner for as long as you like. The system is the same.



Cardinal Numbers and Ordinal Numbers

10. *Cardinal numbers,* or *cardinals,* are numbers that simply answer the question, How many? For example, one, two, three, and so on, are cardinals. *Ordinal numbers,* or *ordinals,* are numbers that are used to show the position of a thing or unit in a series. For example, when we speak of the *third* house or the *sixth* of June, *third* and *sixth* are ordinal numbers. Some cardinals and ordinals are shown in Table 1. Note that, in general, ordinals are formed by adding the letters

TABLE 1

ONE-DIGIT AND TWO-DIGIT NUMBERS

Cardinal Numbers		Ordinal Numbers	
Figures	Names	Names	Simplified Forms
0	zero	. . .	. . .
1	one	first	1st
2	two	second	2nd
3	three	third	3rd
4	four	fourth	4th
5	five	fifth	5th
6	six	sixth	6th
7	seven	seventh	7th
8	eight	eighth	8th
9	nine	ninth	9th
10	ten	tenth	10th
11	eleven	eleventh	11th
12	twelve	twelfth	12th
13	thirteen	thirteenth	13th
14	fourteen	fourteenth	14th
15	fifteen	fifteenth	15th
16	sixteen	sixteenth	16th
17	seventeen	seventeenth	17th
18	eighteen	eighteenth	18th
19	nineteen	nineteenth	19th
20	twenty	twentieth	20th
21	twenty-one	twenty-first	21st
22	twenty-two	twenty-second	22nd
23	twenty-three	twenty-third	23rd
24	twenty-four	twenty-fourth	24th
25	twenty-five	twenty-fifth	25th
26	twenty-six	twenty-sixth	26th
27	twenty-seven	twenty-seventh	27th
28	twenty-eight	twenty-eighth	28th
29	twenty-nine	twenty-ninth	29th
30	thirty	thirtieth	30th
40	forty	fortieth	40th
50	fifty	fiftieth	50th
60	sixty	sixtieth	60th
70	seventy	seventieth	70th
80	eighty	eightieth	80th
90	ninety	ninetieth	90th

"th" to a cardinal, as four*th*, six*th*, ten*th*, and fourteen*th*. But if you examine Table 1 carefully, you will see that there are exceptions to this rule. For example, first, second, third, fifth, and ninth are the ordinals of one, two, three, five, and nine, respectively. If in doubt about how to spell a particular ordinal, consult a dictionary.

Check Your Learning

17. The number that is 1 greater than 9 is _____. (10)

18. The whole number that comes immediately after 49 is _____. (50)

19. The largest two-digit number is _____. (99)

20. The next whole number after 101 is _____. (102)

21. The smallest four-digit number is _____. (1000)

22. The number 10,000 written in words is _____ _____. (ten thousand)

23. Numbers such as one, two, and three that simply answer the question How many? are called _____ numbers. (cardinal)

24. Numbers such as first and fourth, which are used to indicate the position of a thing as a unit in a series, are _____ numbers. (ordinal)

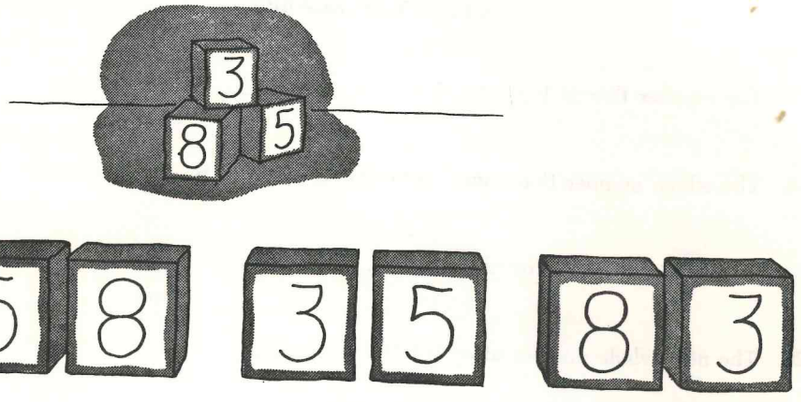
25. In general, an ordinal number is formed by adding the letters _____ to a cardinal number. (th)

Simple and Relative Values

11. The value represented by a figure depends on its position in relation to other figures. Thus figures have *simple values* and *relative values*. The simple value of a figure is the value the figure has when it stands alone. For example, the figure 2 standing alone has a value that is one greater than 1 and one less than 3. But if the figure 1 is placed to the right of the 2, making 21, the first figure (2) no longer has the value it had before. The new value that it has acquired because another figure was placed to its right is called the relative value of the figure. The difference between simple and relative values is illustrated by the blocks in the accompanying sketch. Or let's look at it another way:

If the figure 8 stands alone, thus: 8

The 8 is simply *eight units*, or eight.



But place a 2 to the right of the 8, thus: 82

The 2 is now *two units*, but the 8 has moved one place to the left, so that it is no longer *eight units*. Instead, it is *eight tens*, or *ten times* 8.

Now place a 5 to the right of 82, thus: 825

The 8 is now moved another place to the left, and its value is again increased ten times, or *ten times eight tens*, making *eight hundreds*. At the same time, the 2 is moved one place to the left, and its value is increased to *two tens*.

Now place a 6 to the right of 825, thus: 8256

The 8 is now moved still another place to the left, and its value is increased ten times more, or to *eight thousands*. The 2 is increased to *two hundreds*, and the 5 to *five tens*.

The last number of four figures is read *eight thousand, two hundred fifty-six*.

Check Your Learning

26. The value represented by a figure depends on its _____ in relation to other figures.

(position)

27. The value a figure has when it stands alone is called its _____ value.

(simple)

28. When another figure is placed to the right of the original figure, its new value is called the _____ value of the figure.

(relative)

Importance of Place Value of Digits

12. You began your study of the number system by learning what whole numbers are and how they are used when counting. You saw how whole numbers can be written both in terms of words and in terms of symbols. Before you learn how to use whole numbers to perform such basic operations as addition, subtraction, multiplication, and division, you must first know how to read and write whole numbers.

As you learned earlier, our number system is composed of the ten *figures:* 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. All numbers are combinations of these figures. Thus in a four-digit number like 7852, the digits are 7, 8, 5, and 2. In such a number, each digit has what is called a certain *place value* or *position value*, and the position (place) held by each digit in a number is what gives the digit its value. These place values, reading from right to left in the number, are called *units, tens, hundreds, thousands, ten thousands, hundred thousands, millions* (thousand thousands), *ten millions, hundred millions*, and so on. Thus in our four-digit number 7852, the digit 2 stands in the units place, the 5 in the tens place, the 8 in the hundreds place, and the 7 in the thousands place.

How to Read a Number

13. To read a number then, we state each digit, starting at the extreme left, and give its place value. When reading the number 7852, for example, we merely say seven thousand, eight hundred fifty-two. Strictly speaking, the last two places at the right could be read as 5 tens and 2 units, but this is awkward, and can be shortened to read fifty-two, since fifty means 5 tens. You can see, then, that our method of reading the number 7852 is just a short, convenient way of saying seven times one thousand, plus eight times one hundred, plus five times ten, plus two.

A larger number, such as 27,368,245, is separated into groups of three digits and one group is read at a time. As each group is read, it is given its group name. Thus the number 27,368,245 would be read as twenty-seven million, three hundred sixty-eight thousand, two hundred forty-five.

Grouping of Digits

14. As we just mentioned, large numbers are separated into groups of three digits for greater ease in reading and writing. This is done by counting the digits off from the right and using a comma to separate each group of three. The number ten thousand, for example, is written 10,000. Notice that the three digits at the right end are separated from the rest of the number by a comma. This makes it easier to see that the number is ten thousand than if it were written as 10000.

Similarly, you would find it very difficult to read the number 321672395 without grouping the digits. But beginning with the units digit, which is 5, and separating each group of three digits with a comma, we get 321,672,395. You can see that the number is now much easier to read. Just begin at the left and read each group separately, stating its group name: three hundred twenty-one million, six hundred seventy-two thousand, three hundred ninety-five.

The names of the different groups and places are given in Table 2. You should take a little time right now to memorize the names of the places. Look at Table 2

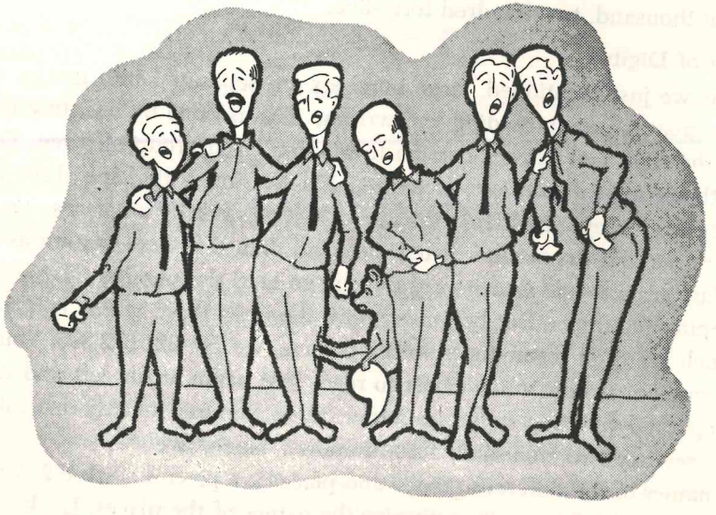
TABLE 2

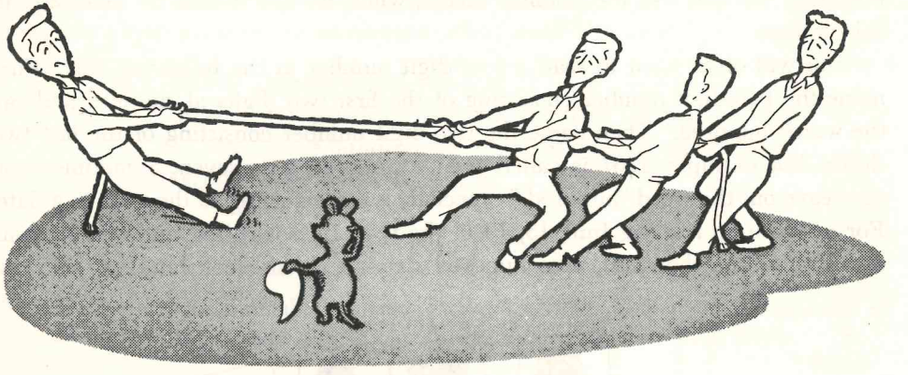
HOW TO READ NUMBERS

Number of Place	Billions Group			Millions Group			Thousands Group			Units Group		
	12th	11th	10th	9th	8th	7th	6th	5th	4th	3rd	2nd	1st
Name of Place	Hundreds of billions	Tens of billions	Billions	Hundreds of millions	Tens of millions	Millions	Hundreds of thousands	Tens of thousands	Thousands	Hundreds	Tens	Units
	1	2	7	9	8	7	7	6	5	4	3	2

and start reading the names of the places from right to left, saying units, tens, hundreds, thousands, and so on, up to hundreds of billions. Then go back to the units group and read the names again. Do this until you have memorized them. Now let's see how your knowledge of Table 2 will help you in reading and writing numbers.

Let's say that after performing some calculations you get an answer of 127987765432. The first thing to do is to mark the answer off with commas, counting in groups of three from the right. Then you have 127,987,765,432. The procedure in reading the result is as follows: *Read each group, or period, from the left, as though it were a separate number, and then state the name of the group after it.* For instance, the first group from the left in the number 127,987,765,432





is read *one hundred twenty-seven.* Then *billion,* the name of the group, is added. Now say *nine hundred eighty-seven million,* which is the second group. The third group is read *seven hundred sixty-five thousand,* and the last group is read simply as *four hundred thirty-two.* Note that the word "unit" has been omitted. Note also that the final *s* in *hundreds, thousands, millions,* and *billions* has also been omitted. The complete figure would thus be read *one hundred twenty-seven billion, nine hundred eighty-seven million, seven hundred sixty-five thousand, four hundred thirty-two.*

Check Your Learning

29. There are _____ figures in our number system.
(ten)

30. All numbers which can be formed are _____ of the ten figures.
(combinations)

31. In a four-digit number such as 7852, the digit 2 stands in the units place, 5 in the tens place, 8 in the hundreds place, and 7 in the _____ place.
(thousands)

32. To read a number, you state each digit, starting at the extreme _____ and give the number its place value.
(left)

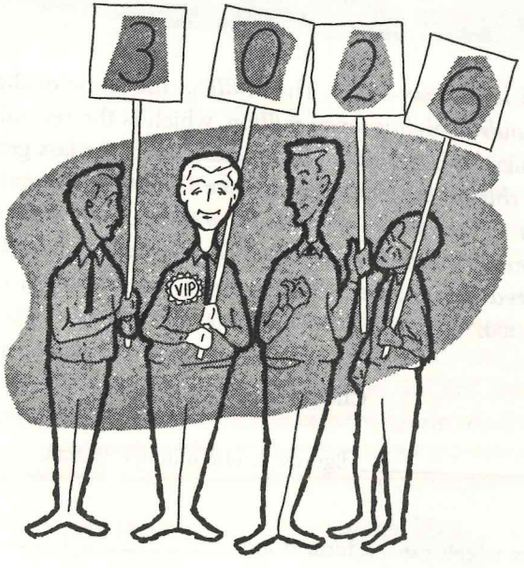
33. There are _____ tens in the number fifty.
(five)

Some Remarks on Four-Digit Numbers

15. It is a common practice to omit the comma if a number contains only four digits, unless there is a special reason for using it. Here, to help you in reading

numbers, we will use the comma except when its use would be awkward in calculations.

You will often want to read a four-digit number in the following way: First name the two-digit number consisting of the first two digits at the left, and say the word "hundred." Then name the two-digit number consisting of the last two digits. For example, 1267 is called twelve hundred sixty-seven. Sometimes you can leave out the word "hundred," especially when referring to the year in a date. For instance, in reading June 15, 1966, the year 1966 is called nineteen sixty-six rather than one thousand, nine hundred sixty-six or nineteen hundred sixty-six.



Use of the Zero

16. The zero, or 0, has no value in itself, because it indicates "none" in answer to the question How many? However, it is necessary in determining the position of other figures. For example, let us consider the number three thousand twenty-six. It would be written 3,026. In this number the zero is very important because without it you would have 326, which is only three hundred twenty-six. As another example, consider the number six thousand four, which would be written 6,004. Here again the zeros are necessary so that the 6 will be in the thousands group. Now let's point off and read a number that has zeros, such as 50609. Counting three digits from the right and placing the comma, we have 50,609. The first group of digits at the left is the thousands group. So in reading the result, the first thing you would say is fifty thousand. The last group at the right is the units group, and you would simply say six hundred nine. The complete number

would thus be read fifty thousand, six hundred nine. That was easy, wasn't it? Here are a few more examples.

10,010	ten thousand, ten
46,203	forty-six thousand, two hundred three
381,089	three hundred eighty-one thousand, eighty-nine
400,000,000	four hundred million

Check Your Learning

34. When you write a number containing more than four digits, you should mark off the number into groups with _____ digits in each group.
(three)

35. Numbers are marked off by using a _____ to separate each group of three digits, counting from the right.
(comma)

36. Writing the number ten million in digits and marking it off correctly gives _____.
(10,000,000)

37. In itself, the digit zero (0) doesn't have any _____.
(value)

Practice Problems

Write your answers to the following problems on scratch paper. Compare your answers with the answers given at the back of this instruction text. Sometime later, write the answers to all the problems once more as a review. These answers are *not* to be submitted to the Schools.

1. Mark off and read the following numbers:
   a) 31072 *b)* 317020 *c)* 1007 *d)* 6051 *e)* 28970093

2. Write the names of the following cardinal numbers:
   a) 6,230 *d)* 9,075 *g)* 14,307
   b) 20,008 *e)* 108,614 *h)* 180,900
   c) 562,341 *f)* 444,444 *i)* 3,600,000

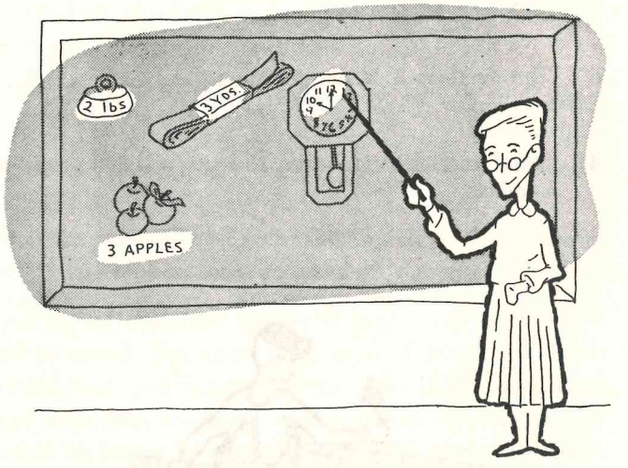
3. Write each of the following numbers in figures:
   a) four thousand, six hundred sixteen
   b) five thousand, two hundred eighty
   c) eleven thousand, forty-seven
   d) thirty thousand, nine hundred three
   e) three hundred thousand
   f) five hundred thirty-four thousand, eight hundred twenty-five
   g) seventeen hundred seventy-six
   h) thirteen million, eight hundred thousand, two hundred

4. *a)* Using figures, write the four numbers that come next in order after 4,998.
   b) Write the three numbers that come next in order after 52,629, again using figures.
   c) Write, in figures, the two numbers that come next in order after 659,999.

Four Basic Processes

17. There are four basic processes, or operations, in arithmetic. They are *addition, subtraction, multiplication,* and *division.* These are basic operations because every calculation depends upon one or more of them. As we mentioned earlier, everyone uses the basic operations of arithmetic. The worker in every kind of employment meets problems that call for figuring. For example, the designer must know the dimensions of a machine part that is to sustain a certain pull or pressure. The foundryman has to calculate the amount (weight) of metal needed to pour a casting. The boiler inspector has to know the steam pressure allowable on a boiler. The engineer needs to do some figuring if he wishes to know the horsepower of an engine. The accountant uses numbers to find such things as bank balances, interest, discounts, taxes, and profit and loss. The electrical engineer or electrician, the radio or television repairman — all must be able to calculate. In fact, it would be very difficult, if not impossible, to name anyone who does not need to know *how to calculate.*

All calculations are made to obtain certain desired information. Some calculations are short and simple; others are long and tedious. However, anyone who



hopes to advance to positions of responsibility and authority must learn how to perform all kinds of calculations rapidly and accurately.

Concrete and Abstract Numbers

18. *Concrete numbers* are numbers that definitely refer to some particular unit or thing, such as 8 men, 15 ft (feet), $265. Be sure to note that each expression — 8 men, 15 ft, $265 — is the concrete number. On the other hand, numbers that are not associated with any particular thing or unit are called *abstract numbers.* For example, the numbers 3, 5, and 26, without reference to any particular thing or unit, are abstract numbers.

When you are solving problems in arithmetic, you must consider the names of the units or things involved in order to know what numbers to add, subtract, multiply, or divide. But while you are performing the actual addition, subtraction, multiplication, or division, you use only the numbers themselves; you ignore the names of the units.

You will see clearly just exactly what we mean by concrete and abstract numbers as you study the illustrative examples in this text. In the more elementary examples, abstract numbers are used; no concrete numbers are involved. However, there are many examples which show you how concrete numbers are used.

Check Your Learning

38. Numbers used to refer to particular units or things are _____ numbers.

(concrete)

39. Numbers that aren't associated with any particular unit or thing are called _____ numbers.

(abstract)

40. The numbers 3, 6, and 72 are _____ numbers.

(abstract)

41. Numbers such as 12 books, 18 coats, and 200 people are called _____ numbers.

(concrete)

Addition

Preparation for Addition

19. Before you go on with your study of this text, ask yourself the following two questions, and be certain that you can answer each one with a definite "Yes."

1. Do I understand thoroughly how the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 are used to form numbers?
2. Do I know how to count, especially when I have to find the number that comes after a number ending in 9?

If you were able to get the correct answers to all the practice problems very easily, you are ready to go on. But if you could not get the answers to some of the problems without a great deal of thought and without looking again at the explanation in the text, you should review the preceding parts of this text carefully at least once more.

Definitions

20. The process of finding a number which is equal in value to the combined value of two or more given numbers is called *addition*. The numbers that are to be added are called *addends*. The number obtained by performing addition is called the *sum*, or *total*. Thus, when the addends 2 and 3 are added, the sum is 5. Likewise, the sum of the addends 4 and 3 is 7. The sign of addition is +, which is

read *plus*. This sign is placed between the numbers that are to be added, to indicate that addition is to be performed. Thus 6 + 4 indicates that the sum of 6 and 4 is to be found, and is read six plus four.

Purpose of Addition

21. Some beginners in arithmetic may say, "I know how to add, but sometimes when a problem comes up I'm not sure whether adding is the proper thing to do." This raises the question, When do you add? Probably the best instruction we can give you on this question is this:

You add when you know some numbers and you want to find the one number you would get if all of these known numbers were counted off, one after the other.

For example, suppose you go to a grocery store and buy a loaf of bread, a quart of milk, and a package of cereal. You know what each of these items costs separately, and to find the total cost, you would add the costs of the individual items. In this case, the cost of each item would be a known number of cents, and the total number of cents could be found by counting off all the known numbers of cents, one after the other.

Check Your Learning

42. The process of finding a number that is equal in value to the combined value of two or more given numbers is called _____.

(adding)

43. In an addition problem, the numbers that are to be added are called _____.

(addends)

44. When two or more numbers are added, the result is called the _____.

(sum)

45. The sign of addition is called the _____ sign.

(plus)

46. The sum of six plus four is _____.

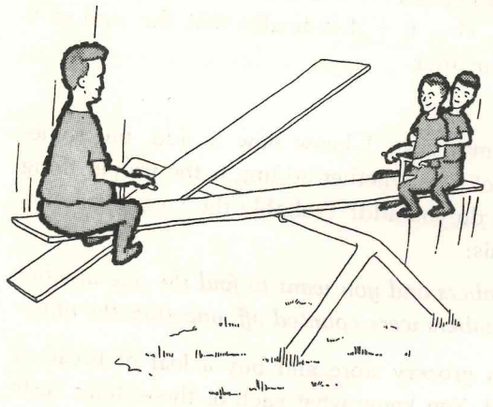
(10 or ten)

47. If you should go to a grocery store and buy a loaf of bread, a quart of milk, and a package of cereal, to find the total cost you would _____ the costs of the individual items.

(add)

Addition Restricted to Like Things

22. Numbers to be added must denote things of the same kind. Thus 6 dollars



can be added to 4 dollars, giving a sum of 10 dollars. But 6 dollars cannot be added to 4 ft, because dollars and feet are not like things.

Basic Principle in Adding Two Numbers

23. By now you should see that the basic principle in adding two numbers may be outlined as follows: Start with one of the given numbers, and count from that number as many units as are indicated by the other given number. For example, to add 3 and 2, start with the number 3 and count two more units. The first unit after 3 is 4, and the second unit after 3 is 5. Therefore, when 3 and 2 are added, the result is 5. The fact that the sum of 3 and 2 is 5 can be shown simply in the following equation:

$$3 + 2 = 5$$

This is read three plus two equals five. It is called an equation because it is a mathematical way of writing the fact that two things are equal to each other. The sign of equality, or equals sign, =, means "equals" or "is equal to." Hence the expression $2 + 3 + 7 = 12$ is read two plus three plus seven equals twelve.

The equals sign is used very often in solving problems. The equals sign *always* means that the value of the numbers to the left of the sign is exactly the same as the value of the number to the right of the sign.

Basic Addition Table

24. You've just seen how you can add numbers by counting. However, you certainly wouldn't want to count like this every time you were required to add numbers, even if they were small ones. To avoid the need for counting when you add, you must memorize the sums of all the possible pairs of numbers from 0 to 9. Addition can be regarded as a shortcut method of counting because a knowledge of these sums makes it possible to add numbers without actually counting. For your convenience in learning these sums, all the possible pairs of numbers and

TABLE 3

BASIC ADDITION TABLES

1+0= 1	2+0= 2	3+0= 3
1+1= 2	2+1= 3	3+1= 4
1+2= 3	2+2= 4	3+2= 5
1+3= 4	2+3= 5	3+3= 6
1+4= 5	2+4= 6	3+4= 7
1+5= 6	2+5= 7	3+5= 8
1+6= 7	2+6= 8	3+6= 9
1+7= 8	2+7= 9	3+7=10
1+8= 9	2+8=10	3+8=11
1+9=10	2+9=11	3+9=12
4+0= 4	5+0= 5	6+0= 6
4+1= 5	5+1= 6	6+1= 7
4+2= 6	5+2= 7	6+2= 8
4+3= 7	5+3= 8	6+3= 9
4+4= 8	5+4= 9	6+4=10
4+5= 9	5+5=10	6+5=11
4+6=10	5+6=11	6+6=12
4+7=11	5+7=12	6+7=13
4+8=12	5+8=13	6+8=14
4+9=13	5+9=14	6+9=15
7+0= 7	8+0= 8	9+0= 9
7+1= 8	8+1= 9	9+1=10
7+2= 9	8+2=10	9+2=11
7+3=10	8+3=11	9+3=12
7+4=11	8+4=12	9+4=13
7+5=12	8+5=13	9+5=14
7+6=13	8+6=14	9+6=15
7+7=14	8+7=15	9+7=16
7+8=15	8+8=16	9+8=17
7+9=16	8+9=17	9+9=18

the correct results are shown in Table 3. This table should be memorized. If you don't know these simple sums well, study Table 3 until you're sure that you'll remember them. Repeat each equation aloud several times so that it will be impressed firmly on your mind.

Note that the sum of two numbers is the same, no matter which of the two numbers is first. For example, $2 + 4 = 6$ and $4 + 2 = 6$. Also, $5 + 8 = 13$ and $8 + 5 = 13$. You must be able to give the sum quickly and correctly, no matter how the numbers are stated.

Be sure to notice, and to remember, that *the sum of any given number and zero is the same as the given number.* Thus $4 + 0 = 4$ and $6 + 0 = 6$. When you add numbers of more than one digit, you will often apply this principle.

Check Your Learning

48. Six books (can, cannot) be added to four pencils.

(cannot)

49. You can add only numbers denoting things of the _____ kind.

(same)

50. The mathematical way of writing the fact that two things are equal to each other is called an _____.

(equation)

51. The sign of equality means _____.

(equals)

52. When you have several given numbers and want to find the one number you would get if all these numbers were counted off, one after the other, you _____ the numbers.

(add)

53. The sum of 5 + 7 (is, is not) the same as the sum of 7 + 5.

(is)

54. The sum of a given number and zero is the same as the _____.

(given number)

Practice Problems

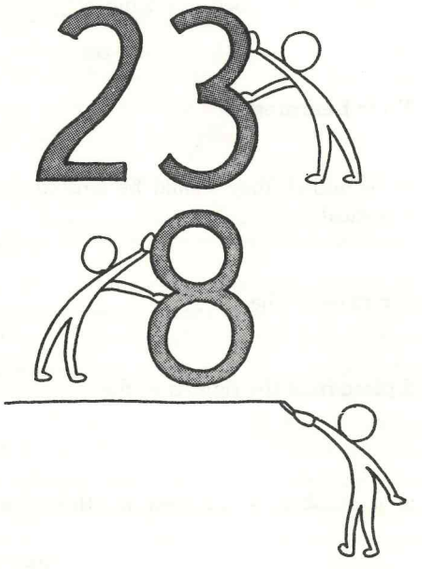
When you feel certain that you know the sums in Table 3, test yourself by doing the practice problems that follow. Without counting or referring to Table 3, write on a piece of paper the sums of the following pairs of numbers. Keep a record of the time it takes you.

3 7	2 9	4 6	2 2	7 5	6 3	1 8	5 5	3 4	7 8
5 0	9 8	1 9	7 6	9 3	1 1	0 3	5 8	2 7	5 4
3 6	7 1	4 2	4 7	5 3	7 7	9 7	1 3	3 9	6 6
3 2	1 6	3 3	9 4	5 7	8 6	8 2	9 5	8 8	6 8
8 5	3 8	4 1	5 6	4 8	6 9	9 2	7 4	8 3	7 9

Now compare your answers with the correct ones given in Table 3 or at the back of this text. If you have made any mistakes, study again whatever parts of Table 3 you are weak in. Sometime later, perhaps in a day or so, do the problems again. Practice until you can get all the answers correct in sixty seconds. When you have mastered this exercise, you will have made important progress in arithmetic.

Preparing Numbers for Adding

25. Up to now, addition has been indicated for the most part by writing the given numbers with a plus sign between them. Also, the equals sign was written after the given numbers, and the sum was placed after that sign. However, when two or more whole numbers are to be added, the usual procedure is to write the



numbers down, one below another, with the right-hand digit of each number directly below the right-hand digit of the number above it, as we did in the set of practice problems you just worked out. This right-hand digit denotes the units place; hence all the digits in the units place will be in the same vertical row, or *column*. The digits in the second place from the right, the tens place, in the numbers to be added will form the second column. The third column from the right will contain all the digits in the hundreds place. If the numbers are larger, columns are added to the left for the thousands place, and so on, as the numbers require.

A horizontal line is drawn below the last number of the group to be added. This line shows that no more numbers are to be included. The sum of the numbers is written below this line.

EXAMPLE 1. Arrange the following numbers in the proper position for adding: 475, 309, 182, and 764.

SOLUTION. Write the numbers one under another, taking care to have all the right-hand digits in the same column. Since each of the numbers contains three digits, there will be three columns, as shown at the right.

```
 475
 309
 182
 764
```

EXAMPLE 2. The following numbers are to be added: 272,130, 67,777, 158,309, and 4,034. Write them down in the proper position for adding.

SOLUTION. The numbers are written as shown at the right, one under another, with the right-hand digits in the same column. Some of these numbers are longer than others, but just as before, the right-hand digits are kept in one column. There are six columns because some of the numbers have six digits.

```
272130
 67777
158309
  4034
```

Check Your Learning

55. When several whole numbers are to be added, they should be written one under the other with the units digits in a vertical _____. (column)

56. The right-hand digit of any number denotes the _____ place. (units)

57. In a number, the digit in the second place from the right is in the _____ place. (tens)

58. The third place from the right in a number is occupied by the digit in the _____ place. (hundreds)

59. In an addition problem, a horizontal line is drawn beneath the _____ number to be added. (last)

60. In an addition problem, the number written below the horizontal line is called the _____. (sum)

Procedure in Addition

26. Before we go into the actual addition of numbers containing three or more digits, such as the ones shown in the examples in the preceding article, we will work out a simpler problem. The procedure for finding the sum of any series of whole numbers can be stated in the form of a rule, as follows:

RULE. *Arrange the numbers so that like digits are in the same column.* That is, you'll have a units column, a tens column, a hundreds column, and so on.

Beginning at the right, add each column separately, and write the sum, if it is less than ten, under the column added.

If the sum of any column is ten or more, write down the units digit only under the column, and carry the tens digit to the next column, where it will be added with that column.

Write the full sum of the last column.

EXAMPLE 1. Add the following numbers: 17, 35, 42, and 3.

SOLUTION. Follow all the instructions just given in the rule for addition. Write the digits in columns, being careful to put each right-hand digit in the units column. The total of the units column is 17, and so you write the 7 below the line in the units place, and carry the 1 to the tens place. This carried digit is shown in smaller type at the top of the tens column. Then add the tens column. Be sure to include the carried digit. The sum is 9, which you write below the line in the tens place. This gives you 97. Ans.

```
 1
17
35
42
 3
97
```

Now let's try a few more difficult problems, using three or more digits.

EXAMPLE 2. Find the sum of the numbers 362, 458, 120, and 876.

SOLUTION. The given numbers are written as at the right, and the total and the carried digits (shown here in smaller type) are shown. The sum of the digits of the given numbers in the units column is 16. The 6 is written in the units place in the total, and the 1 is carried to the tens column. When the digits in the tens column are added, the sum is 21. So the 1 is written in the tens column in the total, and the 2 is carried to the hundreds column. Finally, the sum of the digits in the hundreds column is 18. Since there are no other columns, both digits, the 1 and the 8, are written in the sum, as shown, and the total is 1,816. Ans.

```
 21
362
458
120
876
1816
```

EXAMPLE 3. Find the sum of the numbers 272,130, 67,777, 158,309, and 4,034.

SOLUTION. The complete work would usually be shown as in the arrangement at the right. The digits in small type at the top are those that are carried from the next column at the right. Thus the sum of the digits in the units column is 20, and the 2 is put at the top of the tens column to be added with the digits of the given numbers in that column. You should add the digits in each column yourself, to see that you obtain the correct sums. They are 15 for the tens column, 12 for the hundreds column, 22 for the thousands column, and 20 for the tens of thousands column.

```
 22112
272130
 67777
158309
  4034
502250
```

Where there is a blank space in a column, as in the tens of thousands place in the number 4,034, you simply skip that space or imagine that it is occupied by a zero. Thus the digits to be added in the tens of thousands column are 2 (the carried digit), 7, 6, and 5. Their sum is 20.

For practice, and to make sure your result is correct, you should add the digits in each column both upward and downward. The total is 502,250, as shown. Ans.

EXAMPLE 4. Find the sum of the numbers 425, 36, 9,215, 4, and 907.

SOLUTION. The arrangement at the right shows the usual method of indicating the work. The steps in the solution are the same as those in the preceding two examples. Here the sum of the digits in the units column is 27. Therefore, the 7 is written in the total and the 2 is carried to the tens column, as shown. The sum of the digits in the tens column is only 8, so no digit is carried to the hundreds column. The blank space in the tens column next to the number 4 is simply ignored.

```
  1 2
  425
   36
 9215
    4
  907
10587
```

In the hundreds column there are two blank spaces, and the sum of the given digits 4, 2, and 9 in that column is 15. When the 1 is carried to the thousands column and added to the 9 of the number 9,215 in that column, the result is 10. Hence the sum of the given numbers is 10,587. Ans.

Addition of Long Columns

27. Occasionally, you may have to add so many numbers that the sum of the digits in a column will be greater than 100. For example, suppose that the sum of the digits in the units place is 123. You should write the 3 in the units place of the total, and carry the 12 to the tens column. Similarly, if the sum of the digits in the tens column is 118, write the 8 in the tens place of the total and carry the 11 to the hundreds column.

Practice Problems

Solve the following problems on scratch paper. Then compare your answers with the answers given at the back of this text. Be sure that you can solve these problems quickly and accurately before you go on with your studies. If you have any difficulty, go back and review the explanation in this text.

1. How much is 104 + 203 + 613 + 214?
2. Find the sum of the following four numbers: 1,875; 3,143; 5,826; and 10,832.
3. Add the following four numbers: 865; 2,145; 73; and 40,084.
4. Find the total of the following six numbers: 398; 74,615; 8,200; 411,627; 58,239; and 6,074.
5. Add two hundred thirty-six thousand, eight hundred eighty-five; one hundred eighteen thousand, nine hundred twenty-six; thirty-five thousand, thirty-six; and two hundred forty-five thousand, seven hundred eighteen.
6. Add eight million, three hundred twenty-three thousand, seven hundred ninety-five; sixteen million, two hundred seventy-nine thousand, three hundred sixty-four; eighty-two million, seven hundred fifteen thousand, four hundred twelve; and eighteen million, one hundred twelve thousand, seven hundred ninety-six.

Horizontal Addition

28. Although the usual procedure is to write the numbers in columns and add vertically, it is sometimes convenient to add from left to right, or horizontally.

For instance, suppose that four shops are making a certain part of a machine, and that a record of the number of parts made by each shop for six days is kept in the form of a table, as follows:

	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Total
Shop 1	2,763	2,681	2,915	2,820	2,794	2,808	16,781
Shop 2	3,510	3,466	3,287	3,461	3,385	3,527	20,636
Shop 3	1,806	2,012	1,857	1,951	2,006	1,898	11,530
Shop 4	2,351	2,308	2,296	2,174	2,256	2,402	13,787
Total	10,430	10,467	10,355	10,406	10,441	10,635	62,734

The output of all four shops for each day is found by adding the six columns in the usual way, and writing the totals below the line. However, the output of each shop for the week must also be found, and this is done by adding the six numbers in each horizontal row. Of course, you could copy off the six numbers and arrange them one below the other so that they could be added in the usual way. But that would take time, and errors might be made in rewriting the numbers. We can save time and avoid the chance of errors due to copying the numbers by adding each row from left to right. To do this, start with the last digit, 3, of the first number, 2,763, in the first row, and to this 3 add the last digit of each of the other numbers in the same horizontal row. The sum of these digits is 3 + 1 + 5 + 0 + 4 + 8, or 21. The 1 is written down in the units place under *Total*, and the 2 is carried over and added to the digits in the tens place in the numbers of the first row, giving 2 + 6 + 8 + 1 + 2 + 9 + 0, or 28. The 8 is written down in the tens place next to the 1 in the Total column, and the 2 is carried over to be added to the digits in the hundreds column. That sum is 2 + 7 + 6 + 9 + 8 + 7 + 8, or 47; so the 7 is put down next to the 8 in the hundreds place in the Total column, and the 4 is carried over and added to the digits in the first position – the thousands place – in the six numbers. This sum is 4 + 2 + 2 + 2 + 2 + 2 + 2, or 16, and 16 is written next to the 7 in the Total column, giving 16,781 as the total output of Shop 1 for six days. The output of each of the other shops is found in the same way.

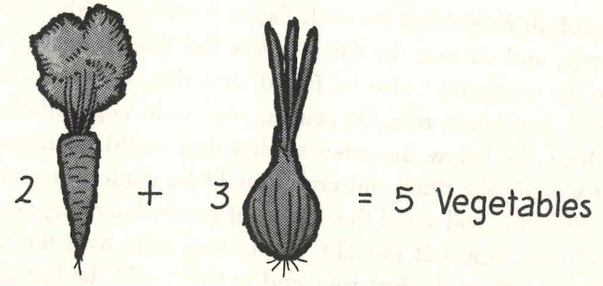
In adding numbers horizontally, you must be very careful not to add digits from the wrong places in the numbers. All digits in the units place are added, then all digits in the tens place, and so on to the last place at the left.

If the weekly outputs of the four shops are added vertically, the total output for the week is found to be 62,734. If the six daily totals below the line are added horizontally, their sum will also be 62,734. It is obvious that the sum of the weekly outputs of the four shops must equal the sum of the six daily outputs. If the sum of the four totals at the right is not the same as the sum of the six daily outputs, there has been a mistake in addition. Thus, adding a group of numbers in two directions, as in the example just given, serves as a check on the correctness of the addition.

Practice Problems

Solve each of the following problems without rewriting the given numbers one below another:

1. $31 + 10 + 53 = ?$
2. $18 + 32 + 25 = ?$
3. $73 + 69 + 24 + 48 + 50 = ?$
4. $99 + 37 + 60 + 75 + 26 = ?$



Adding Concrete Numbers

29. In many problems you will not be told what numbers you are to add. To choose the right numbers, you will often have to consider concrete numbers. Do you remember what concrete numbers are? They are numbers that definitely refer to some particular unit or thing — 2 houses, 12 apples, 100 soldiers.

As we mentioned earlier, only like things can be added. Maybe we should clarify that expression "like things" a little. *Like* things may or may not be the *same* things. As an example of two numbers associated with like things that *are* the same thing, consider the expressions \$10 and \$5. If you have \$10 and a friend gives you \$5, you can find out how many dollars you have by adding 5 to 10. But before you can add, you have to be sure that the 10 and the 5 are associated with like things. Since both are dollar amounts, you can go ahead with the addition. In adding, ignore the dollar sign and say that $10 + 5 = 15$. When you have finished adding, place the dollar sign in front of the result. Then you have \$15.

As another example of two numbers associated with like things that are the same thing, take the expressions 4 ft and 3 ft. If you need two pieces of wire, one 4 ft long and the other 3 ft long, you can find the total length needed by adding 4 and 3. The total length would be 7 ft.

Now let's see how things that are *not* the same may be like things. This is possible because, in deciding whether numbers apply to like things, we can consider broader classes. For example, if there are 10 men and 6 women in a group, you can find the total number of *people* in the group by adding the 10 and the 6. Another example: If 250 passenger cars and 42 trucks passed a certain point in an hour, you can get the total number of *motor vehicles* that passed the point by adding the 250 and the 42. Notice that in each of these examples the like things, people and motor vehicles, include things that are not the *same*. Thus in the one instance, "people" includes men and women, and in the other, "motor vehicles" applies to cars and trucks.

You may have to find the sum of two distances, one given in feet and the other in inches. You cannot add a number of feet to a number of inches. But you can add the two distances if you first express them in the same unit. In other words, before you add, you must express both distances either in inches or in feet.

Check Your Learning

61. Arrange the numbers to be added one below the other so that the _____ having the same place value are in the same column.

(digits)

62. Beginning at the right, add each column separately, writing the sum of each column, if it is less than _____, directly below that column.

(10)

63. If the sum of the units column is ten or more, write down the units digit *only* under the column, and carry the _____ digit to the next column, where it will be added in with that column.

(tens)

64. In an addition problem, write the full _____ of the extreme left-hand column.

(sum)

65. A number of feet and a number of inches (can, cannot) be added together.

(cannot)

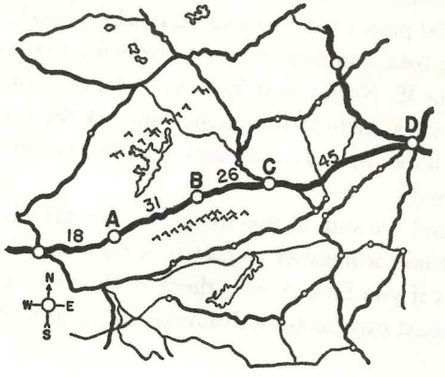
66. You (can, cannot) add 35 bottles to 13 boxes to find the total number of containers.

(can)

Applications of Addition

30. Now let's consider a few problems where concrete numbers are added.

EXAMPLE 1. You are planning an automobile trip, and so you consult a road map. To get where you want to go, you must pass through three towns, A, B, and C. The distance on the map from your home to A is 18 miles, that from A to B is 31 miles, that from B to C is 26 miles, and that from C to your destination is 45 miles. How many miles will you drive?



SOLUTION. You can find the total length of the trip by adding the distances for the parts of the trip. In this example you must find the sum of 18 miles, 31 miles, 26 miles, and 45 miles. Since the same unit, miles, is used for all these distances, you can add the given numbers and can call the sum a number of miles. Thus $18 + 31 + 26 + 45 = 120$, and the required distance is 120 miles. Ans.

EXAMPLE 2. On a certain day, the amounts of a salesman's orders were \$168, \$573, \$54, \$1,010, \$98, and \$404. What was the total amount of these orders?

SOLUTION. The same unit, dollars, is used for each of the given amounts of money. Therefore, the required total can be found by adding these numbers and calling their sum a number of dollars. Write the numbers one below the other, and add them in the usual manner. The correct total is \$2,307. Ans.

Although the details of the addition are not shown here, you should do the work. First add the digits in each column by starting at the bottom, and then verify your results by starting at the top and adding down.

EXAMPLE 3. On a certain day, the numbers of people who attended National League baseball games in four cities were as follows: 42,865, 33,602, 9,470, and 25,589. On the same day, the numbers of people who were at American League games in four cities were 12,511, 19,478, 50,250, and 32,906. *a)* What was the total attendance at the National League games? *b)* What was the total attendance at the American League games? *c)* What was the total attendance for both leagues?

SOLUTION. *a)* Each given number refers to people, and so these numbers can be added to find another number of people. To get the total number of people who saw the National League games, you should add the four numbers in the first group. The sum is shown here, but the digits that are carried from the several columns are not indicated. The total for the National League is 111,526. Ans.

b) The total attendance at the American League games is the sum of the four numbers in the second group. As shown here, this total is 115,145. Ans.

c) It would be possible to find the total attendance for both leagues by adding the eight given numbers in the usual manner. However, since you have already found the sum of the first four numbers and also the sum of the last four numbers, you should simply add 111,526 and 115,145, as indicated here. The total is 226,671. Ans.

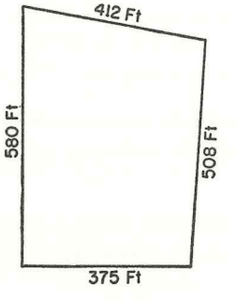
National League	*American League*	*Both Leagues*
42,865	12,511	111,526
33,602	19,478	115,145
9,470	50,250	226,671
25,589	32,906	Ans. *(c)*
111,526	115,145	
Ans. *(a)*	Ans. *(b)*	

Practice Problems

These practice problems will give you a chance to test your understanding of concrete numbers. Take your time and solve them carefully. Check your answers against the answers in the back of this text. Remember, if you have any trouble, review the text material until you have a clear understanding of it.

1. A truck is to be loaded with three castings having the following weights: 3,265 lb (pounds), 1,092 lb, and 748 lb. What is the total weight of the castings?

2. The numbers of parts produced by a shop during the first six months of a year were as follows: January, 8,502; February, 8,748; March, 9,215; April, 9,770; May, 10,269; and June, 12,184. What was the total production for the six months?



3. A farmer wants to build a fence around a piece of land having four sides, and shaped as shown. The lengths of the sides are 375 ft, 580 ft, 412 ft, and 508 ft. Find the total length of fence needed.

4. An airliner travels the following distances between cities: 583 miles, 702 miles, 1,240 miles, 675 miles, and 829 miles. What is the total number of miles it flies?

5. A professional football team played six games on its home field. The numbers of people at these games were 35,824; 28,902; 33,580; 19,677; 26,396; and 40,059. What was the total attendance for the six games?

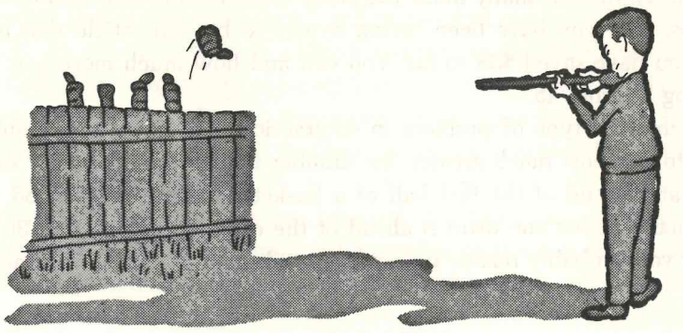
6. The numbers of motor vehicles that crossed a bridge in one week were as follows: on Monday, 167,483; on Tuesday, 78,296; on Wednesday, 95,010; on Thursday, 50,664; on Friday, 112,927; on Saturday, 204,379; and on Sunday, 236,508. How many vehicles crossed the bridge in the week?

7. A pump operated 2 hr (hours) and 45 min (minutes) to empty a tank filled with water. The meter readings showed that 4,200 gal (gallons) were removed during the first hour, 5,420 during the second hour, and 3,600 during the last 45 min. How many gallons of water did the tank contain originally?

8. A heating, ventilating, and plumbing firm has completed three contracts. The payment received on the first contract was $2,560; on the second, $3,125; and on the third, $2,850. How much was received in all?

9. *a)* Find the sum of each of the following three columns, adding *vertically.*
b) Add each row of numbers *horizontally.*
c) Add the results of the horizontal addition, as was explained in Art. 28.

4,568	15,431	7,386
7,391	29,685	45,371
7,854	73,648	13,764
53,469	34,519	9,887
13,470	78,234	64,348
58,143	7,843	14,627



Subtraction

Definitions

31. The process of finding how much is left after one number has been taken away from another is called *subtraction.* The number from which another number is to be subtracted is called the *minuend,* which is the larger of the two numbers. The number which is taken away from the larger one is called the *subtrahend.* The result obtained by subtracting is called the *difference.* Thus, if 4 is subtracted (taken away) from 7, 4 is the subtrahend and 7 is the minuend. The amount remaining after the subtraction has been performed, which is 3, is the difference.

Although you may sometimes hear the result of subtraction referred to as the "remainder," in this text it will be called the "difference." The word "remainder" will be used only in division to indicate that part of the dividend which does not contain the divisor even once. This will be explained more fully in the section of the text dealing with division.

The sign which indicates that one number is to be subtracted from another is −, which is read *minus.* This sign is placed between the two numbers involved in the subtraction, and since this sign means "decreased by," the larger number must be placed in front of the sign and the smaller number must follow it. For example, 12 − 7, which is read twelve minus seven, indicates that 7 is to be subtracted from 12. The difference is 5; that is, $12 - 7 = 5$.

Purpose of Subtraction

32. At this point it is natural to ask, "When do I subtract?" You should always subtract when you have a known number of things before you give or take some away, and you want to know how many will be left. For example, suppose that you have a dollar, or 100 cents, and that you buy a few groceries that cost 66 cents. To find how many cents will be left, or how many cents you should get back from the clerk, you subtract 66 from 100.

You can also use subtraction when you have a known number of things and

want to know how many more you need to have a desired number. For example, suppose that you have been saving money to buy an article that costs \$45, and that you have saved \$38 so far. You can find how much more you need by subtracting 38 from 45.

In another type of problem in subtraction, you know two numbers and you want to find how much greater one number is than the other. For example, if the score at the end of the first half of a basketball game is 44 to 36, you can find how many points one team is ahead of the other by subtracting 36 from 44.

As you probably realize by now, only like things can be subtracted.

Check Your Learning

67. The process of finding how much is left after one number is taken from another number is called _____.

(subtraction)

68. A number from which another number is to be subtracted is called the _____.

(minuend)

69. In subtraction, the number which is taken away is called the _____.

(subtrahend)

70. The result obtained by subtracting one number from another number is called the _____.

(difference)

71. The sign used to indicate that one number is to be subtracted from another number is called a _____ sign.

(minus)

72. In a subtraction problem, the (larger, smaller) number is placed before the minus sign and the (larger, smaller) number is placed after it.

(larger) (smaller)

73. The minus sign means _____ _____.

(decreased by)

74. You must always _____ when you have a known number of things before you give or take some away and you want to know how many are left.

(subtract)

Basic Principle of Subtraction

33. Subtraction is the reverse of addition. A number can be added to another number by counting from either number in the direction in which numbers increase. For example, to add 5 and 2, start at the number 5 and count *ahead* two

units. The number that is two *more* than 5 is 7. A number can be subtracted from another number by counting from the larger number in the direction in which numbers decrease. For example, to subtract 2 from 5, start at the number 5 and count *back* two units, or toward zero. The number that is one *less* than 5 is 4, and the number that is two less than 5 is 3. Thus the result obtained by subtracting 2 from 5 is 3, or the difference is 3.

In subtraction only two numbers can be used at a time; one number is subtrated from the other. Also, in arithmetic, the smaller of the two given numbers must always be subtracted from the larger number. For instance, you can subtract 2 from 5, but you should not try to subtract 5 from 2. You should remember that $2 - 5$ is not the same as $5 - 2$. When subtraction is indicated by writing two numbers with a minus sign between them, the smaller number must always follow the minus sign.

General Remarks on Subtraction

34. Considering subtraction as the reverse of addition, you can subtract by asking yourseclf, "What number must be added to the smaller number to get the larger number?" Obviously, the number that must be added can be found from Table 3, the addition table. Therefore, you don't need a special table for subtraction. For example, you know that $9 + 7 = 16$. If the operation is reversed, and 9 is taken from 16, 7 is left. Or if 7 is taken from 16, 9 is left. As you can see, to be able to subtract you must first be able to add!

Another way of considering subtraction is to think of what number must be added to the smaller number to give the larger number. For example, for $16 - 7$ you think of what number you must add to 7 to give 16. The number is 9, because $9 + 7 = 16$. Likewise, for $16 - 9$ you think of 7, because 7 is the number that must be added to 9 in order to get 16.

You should study Table 3 carefully until, without looking at the table, or counting, you can give the result of any subtraction that can be found from the table. Note that when any number is subtracted from the same number, the difference is zero. Thus $4 - 4 = 0$ and $6 - 6 = 0$. Also, when zero is subtracted from any number, the difference is that number. For instance, $3 - 0 = 3$.

When you want to subtract one number from another, you may show the work in this way: Write the larger number first, put the smaller number under it (as in addition), and draw a line below the smaller number. Then write the difference beneath the line, like this:

minuend	8	12	13
subtrahend	6	8	5
difference	2	4	8

It will be a great help if, in odd moments, you mentally reverse the addition table and use it as a subtraction table. For instance, instead of saying 6 and 5 is 11,

ask, 6 from 11 is how much? 9 from 17 is what? Practice in this manner, using different values, and in a short time the right answer will present itself without any mental effort whatever on your part. You should be able to find the difference between two simple numbers without writing the numbers one below the other. You should see that $8 - 3 = 5$ or that $13 - 8 = 5$ when subtraction is indicated in this way.

Check Your Learning

75. Subtraction is the process which is the reverse of _____.

(addition)

76. In subtraction you can use only _____ number(s) at a time.

(two)

77. When you subtract 0 from 6, your answer is _____.

(6)

78. When the minuend and the subtrahend have the same value, the difference is equal to _____.

(zero)

Now try the following problems.

Practice Problems

1. Find the difference between the two given numbers in each of the following cases. Without looking at Table 3, write your answers on a sheet of paper. Then compare your results with the numbers obtained by using Table 3 or with the correct answers given at the back of this text.

(a)	*(b)*	*(c)*	*(d)*	*(e)*	*(f)*	*(g)*	*(h)*	*(i)*	*(j)*
5	8	7	7	9	6	4	9	2	4
4	6	1	5	3	6	2	8	1	0

(k)	*(l)*	*(m)*	*(n)*	*(o)*	*(p)*	*(q)*	*(r)*	*(s)*	*(t)*
10	14	14	17	13	11	12	16	10	11
6	6	9	8	5	8	4	8	5	2

2. Find the result in each of the following cases:

a) $8 - 7 = ?$
b) $16 - 7 = ?$
c) $12 - 9 = ?$
d) $4 - 1 = ?$
e) $6 - 0 = ?$
f) $11 - 6 = ?$
g) $5 - 5 = ?$
h) $13 - 8 = ?$

3. Find the value that should be written in place of the question mark in the following examples:

a) $9 + ? = 15$
b) $7 + ? = 16$
c) $8 + ? = 17$
d) $5 + ? = 13$
e) $8 + ? = 13$
f) $9 + ? = 17$
g) $7 + ? = 15$
h) $5 + ? = 11$

4. John had $7 and his father gave him $5. The next day John spent $8. How much did he have left?

5. A farmer had 14 cows. In May he sold 5 cows and in June he sold 3 cows. How many cows did he have left?

Subtracting Numbers of Several Digits

35. The simple examples of subtraction given in the preceding article illustrate the principle which is to be used in all problems involving the operation of subtraction. Whether a number has three digits or four digits makes no difference in the process. Problems involving numbers having several digits are solved just like the ones in which the numbers consist of only one digit. Let's clarify this with a few examples.

EXAMPLE 1. Subtract 3,425 from 7,568.

SOLUTION. The numbers are written as for addition as shown at the right, and the line and the difference are placed below them. Note that in this example each digit in the minuend is greater than the digit just below it in the subtrahend. That is, 8 is greater than 5; 6 is greater than 2; 5 is greater than 4; and 7 is greater than 3. Therefore, to obtain the difference, you need only subtract each digit of the subtrahend from the digit in the same column in the minuend. In subtracting, as in adding, you should always start with the digits in the units place and go to the other columns in turn from right to left.

minuend	7568
subtrahend	3425
difference	4143

The digits in the difference are found as follows: $8 - 5 = 3$; $6 - 2 = 4$; $5 - 4 = 1$; $7 - 3 = 4$. The required difference is therefore 4,143. Ans.

EXAMPLE 2. Find the difference between 856 and 402.

SOLUTION. The given numbers and the difference are shown in the arrangement at the right. The procedure is similar to that followed in example 1. In this case, however, there is a zero in the subtrahend. Since $5 - 0 = 5$, the digit in the tens place of the difference is 5. Thus the difference is 454. Ans.

minuend	856
subtrahend	402
difference	454

EXAMPLE 3. What is the difference between 3,869 and 365?

SOLUTION. As shown in the arrangement at the right, the given numbers are written as for addition, with the smaller number below the larger one. To find the digit of the difference in the units place, subtract 5 from 9; you get 4. To find the digit in the tens place, subtract 6 from 6; you get 0. To find the digit in the

minuend	3869
subtrahend	365
difference	3504

hundreds place, subtract 3 from 8; you get 5. Since there is no digit in the thousands place in the subtrahend, the digit 3 in the thousands place of the minuend is brought down to the difference. Hence the required difference is 3,504. Ans.

Borrowing

36. In each of the preceding examples, no digit in the subtrahend was greater than the digit directly above it in the minuend. Often, however, some of the digits in the subtrahend are greater than the digits in the corresponding places in the minuend. In order to subtract when this happens, we must do something new. This new procedure is known as borrowing.

For example, suppose that you must subtract 26 from 94. We'll show the work in detail by breaking it down into three steps:

	Step 1	Step 2	Step 3
minuend	94	80 + 14	94
subtrahend	26	20 + 6	26
difference		60 8	68

Step 1. First the numbers are written one below the other in the usual manner. Now, according to the procedure used in the preceding examples, the 6 in the units place of the subtrahend should be subtracted from the 4 in the units place in the minuend as you have learned. However, in arithmetic, a larger number cannot be subtracted from a smaller number. But we know that 26 can be subtracted from 94 because 94 is larger than 26.

Step 2. So this is what we do: Instead of treating 94 as 90 + 4, we borrow ten from the 90 and consider it as 80 + 14. Thus 6 is subtracted from 14, and 20 is subtracted from 80. As you know, $14 - 6 = 8$ and $80 - 20 = 60$.

Step 3. Since $60 + 8 = 68$, the difference between 94 and 26 is 68, as indicated in Step 3.

We have used three steps to illustrate the procedure of borrowing. But when you do a subtraction problem, write down the given numbers, do the borrowing *mentally*, and write down the difference.

The procedure for subtracting can be stated in the form of a rule, as follows:

RULE. *Write the subtrahend under the minuend so that like digits are in the same column.*

Beginning at the right, subtract each digit of the subtrahend from the corresponding digit of the minuend, and write the result beneath.

If a digit in the subtrahend is greater than the corresponding digit in the minuend, increase the digit in the minuend by 10 and subtract; then reduce by 1 the digit immediately to the left in the minuend, and subtract as before.

The following examples illustrate the procedure when borrowing is necessary.

EXAMPLE 1. Subtract 844 from 6,173.

SOLUTION. In the arrangement at the right, the numbers involved in the subtraction are written one below the other, with the difference written under them. Before the digit 4 in the units place in 844 can be subtracted, the 3 in 6,173 must be increased by 10. (To do this, 1 is borrowed from the 7 in the tens column. This 1 is equal to 10 units.) Then the digit in the units place in the difference is found by subtracting 4 from 13, which gives 9. Also, since the 7 in 6,173 was reduced by 1, the digit in the tens place in the difference is found by subtracting 4 from 6. This digit is 2.

	5 6
minuend	6173
subtrahend	844
difference	5329

The next step is to subtract 8 from 1. Since 8 is greater than 1, the 1 must be increased by 10. (To do this, 1 is borrowed from the 6 in the thousands column. This 1 is equal to 10 hundreds.) Hence, the digit in the hundreds place in the difference is found by subtracting 8 from 11, which gives 3. Finally, since the 6 was reduced by 1, 5 is written in the thousands place in the difference. The complete difference is 5,329. Ans.

EXAMPLE 2. Subtract 8,763 from 20,000.

SOLUTION. Write down the numbers in the usual manner, as shown at the right. Since 3 units cannot be subtracted from 0 units, we must add 10 to the units column, and to do this, we would decrease the tens column by 1 (borrow 1 from the tens column). But the tens column in the minuend is zero. So are the hundreds column and the thousands column. The only column we can begin to borrow from is the tens-of-thousands column. So we borrow 1 from the tens-of-thousands column. That is, we take 1 ten thousand, or 10 thousands, from the 2 and add it to the 0 in the thousands column, giving $10 + 0 = 10$ thousand, and leaving a difference of $2 - 1 = 1$ ten thousand. But it is necessary to continue the borrowing and to take 1 thousand, or 10 hundreds, away from the 10 thousands, and add it to the hundreds column, giving $10 + 0 =$ ten hundreds, and leaving a difference of $10 - 1 = 9$ thousands. But 1 hundred, or 10 tens, must be borrowed from the hundreds to add to 0 tens, giving $10 + 0 = 10$ tens, and leaving $10 - 1 = 9$ hundreds.

minuend	20000
subtrahend	8763
difference	11237

Finally, 1 ten, or 10 units, must be borrowed from the 10 tens to be added to the 0 unit, giving $10 + 0 = 10$ units, and leaving 9 tens. The minuend might now be written as shown at the right. Here the 10 in small figures represents the 10 units, or 1 ten, borrowed from the tens place; so $19{,}990^{10}$ has the same value as 20,000. The subtraction may now be readily performed, as shown. The difference is 11,237. Ans.

	10
minuend	19990
subtrahend	8763
difference	11237

Practice Problems

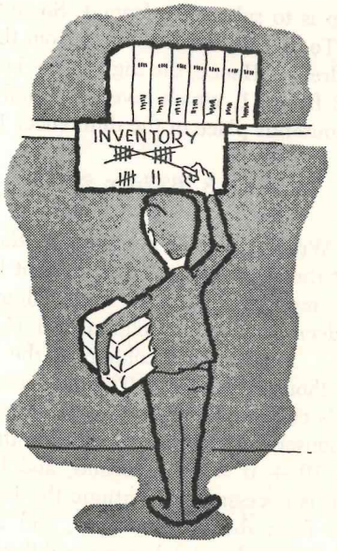
1. Find the difference between the two given numbers in each of the following subtractions and check your results by comparing them with the answers in the back of the text.

(a)	*(b)*	*(c)*	*(d)*
379	2645	94276	53714
126	2023	62557	5824

2. Determine the results of the following subtractions:

a) 542 − 464 = ? *c*) 20804 − 10408 = ?
b) 4319 − 726 = ? *d*) 1000 − 718 = ?

3. Find the difference between:
   a) thirty-nine thousand, four hundred fifty-five and thirty-one thousand, five hundred sixty-seven.
   b) fifty-five thousand and fifty-one thousand, eighty-two.



Applications of Subtraction

37. In each of the preceding examples and problems on subtraction, you were told to subtract a certain number from another number. You were told what numbers to use. But in a practical problem, you will have to decide when you must subtract and what numbers to use. The following examples and solutions should help you learn how to make the correct decisions.

EXAMPLE 1. At the beginning of the month there were 2,000 books of a certain kind in stock. During the month 346 of these books were sold. How many of the books were left in stock at the end of the month?

SOLUTION. The number of books left is the difference between the number on hand at the beginning of the month and the number sold. Thus it is necessary to subtract 346 from 2,000. As shown at the right, the difference is 1,654. Ans.

```
 2000
  346
 1654
```

EXAMPLE 2. A quantity of gasoline was taken from a tank. Before any gasoline was removed, the tank contained 4,253 gal. Afterward it contained 3,518 gal. How much gasoline was taken out?

SOLUTION. The amount removed is the difference between the amount originally in the tank and the amount that remained in it. Therefore, 3,518 should be subtracted from 4,253. Since 4,253 − 3,518 = 735, as shown at the right, it follows that 735 gal were removed. Ans.

```
 4253
 3518
  735
```

EXAMPLE 3. At the beginning of a trip the reading of an automobile speedometer was 13,815 miles. If the reading at the end of the trip was 14,602 miles, how many miles were covered on the trip?

SOLUTION. The distance traveled is the difference between the speedometer reading at the end of the trip and the reading at the beginning of the trip. This difference is found as shown at the right. Therefore, the length of the trip was 787 miles. Ans.

```
 14602
 13815
   787
```

Check Your Learning

79. If a digit in the subtrahend is greater than the digit above it in the minuend, you must _____ before you subtract.

(borrow)

80. To subtract, you begin at the right and subtract each digit of the subtrahend from the corresponding digit of the _____ and write the result beneath the horizontal line.

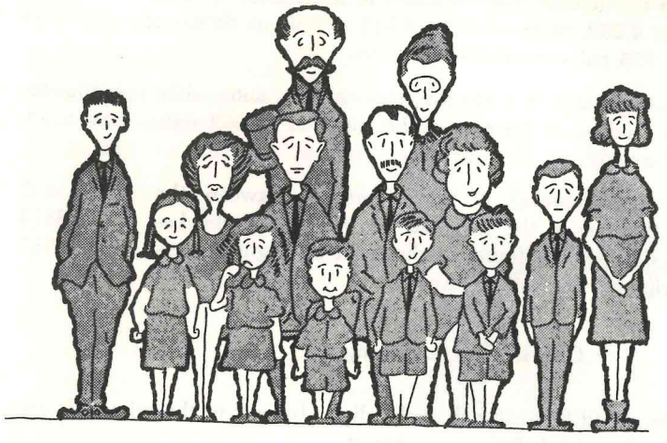
(minuend)

81. If a digit in the subtrahend is greater than the digit above it in the minuend, increase the digit in the minuend by _____ and subtract; then reduce by 1 the digit immediately to the left in the minuend and subtract.

(10)

Practice Problems

1. The score of a basketball game was 108 to 73. By how many points did the winning team beat the losing team?
2. A merchant bought goods for $12,675 and sold them for $15,744. What was the profit?
3. A tank contained 5,000 gal of oil before 500 gal was removed. How many gallons were left in the tank?
4. A railroad car that weighed 48,000 lb when empty was loaded with stone. If the weight of the loaded car with the stone in it was 136,240 lb, how much stone was put in the car?
5. A man had $15,000, but after paying off his debts and giving $3,185 to relatives and charitable organizations, he had only $9,160 left. What was the amount of his debts?
6. If the distance from the earth to the moon is 240,000 miles and the distance from the earth to the sun is 94,000,000 miles, how much farther from the earth is the sun than the moon?



Multiplication

What Multiplication Is

38. Multiplication is simply a shortcut for obtaining the sum of equal numbers. (Don't forget that *sum* implies addition.) To multiply a number, we add it to itself a certain required number of times. Thus, if we wish to find the result of multiplying 5 by 3, we add the number 5 to itself three times. It is easy to see that $5 + 5 + 5 = 15$, but suppose that you want the sum of 21,897 used 87 times. To write 21,897 eighty-seven times and then find the sum would be a long and tedious process. Multiplication is the method that makes such tedious additions unnecessary.

Definitions

39. The number that is to be multipled is called the *multiplicand*. The number that shows how many times the multiplicand is to be taken, or the number by which it is to be multipled, is called the *multiplier*. The sign for multiplication is $\times$, which is written between the numbers that are to be multiplied together, as, for example, 5×4 or 8×7. It is read *times* or *multiplied by*. Thus 7×6 is read 7 times 6 or 7 multiplied by 6.

The result of the multiplication of two numbers is called the *product*. Thus, $5 \times 9 = 45$, 5 is the multiplicand, 9 is the multiplier, and 45 is the product. Several numbers may be used in a multiplication, as, for example, $4 \times 5 \times 3 \times 2 = 120$. It makes no difference in what order the numbers are multiplied; the product will still be the same: that is, $2 \times 3 \times 4 \times 5 = 120$; $5 \times 4 \times 3 \times 2 = 120$; and finally $3 \times 2 \times 5 \times 4 = 120$.

The two or more numbers that are multiplied together to give a product are

TABLE 4

BASIC MULTIPLICATION TABLE

ZERO × ANY NUMBER = ZERO

1× 1= 1	2× 1= 2	3× 1= 3	4× 1= 4
1× 2= 2	2× 2= 4	3× 2= 6	4× 2= 8
1× 3= 3	2× 3= 6	3× 3= 9	4× 3= 12
1× 4= 4	2× 4= 8	3× 4= 12	4× 4= 16
1× 5= 5	2× 5= 10	3× 5= 15	4× 5= 20
1× 6= 6	2× 6= 12	3× 6= 18	4× 6= 24
1× 7= 7	2× 7= 14	3× 7= 21	4× 7= 28
1× 8= 8	2× 8= 16	3× 8= 24	4× 8= 32
1× 9= 9	2× 9= 18	3× 9= 27	4× 9= 36
1×10= 10	2×10= 20	3×10= 30	4×10= 40
1×11= 11	2×11= 22	3×11= 33	4×11= 44
1×12= 12	2×12= 24	3×12= 36	4×12= 48
5× 1= 5	6× 1= 6	7× 1= 7	8× 1= 8
5× 2= 10	6× 2= 12	7× 2= 14	8× 2= 16
5× 3= 15	6× 3= 18	7× 3= 21	8× 3= 24
5× 4= 20	6× 4= 24	7× 4= 28	8× 4= 32
5× 5= 25	6× 5= 30	7× 5= 35	8× 5= 40
5× 6= 30	6× 6= 36	7× 6= 42	8× 6= 48
5× 7= 35	6× 7= 42	7× 7= 49	8× 7= 56
5× 8= 40	6× 8= 48	7× 8= 56	8× 8= 64
5× 9= 45	6× 9= 54	7× 9= 63	8× 9= 72
5×10= 50	6×10= 60	7×10= 70	8×10= 80
5×11= 55	6×11= 66	7×11= 77	8×11= 88
5×12= 60	6×12= 72	7×12= 84	8×12= 96
9× 1= 9	10× 1= 10	11× 1= 11	12× 1= 12
9× 2= 18	10× 2= 20	11× 2= 22	12× 2= 24
9× 3= 27	10× 3= 30	11× 3= 33	12× 3= 36
9× 4= 36	10× 4= 40	11× 4= 44	12× 4= 48
9× 5= 45	10× 5= 50	11× 5= 55	12× 5= 60
9× 6= 54	10× 6= 60	11× 6= 66	12× 6= 72
9× 7= 63	10× 7= 70	11× 7= 77	12× 7= 84
9× 8= 72	10× 8= 80	11× 8= 88	12× 8= 96
9× 9= 81	10× 9= 90	11× 9= 99	12× 9=108
9×10= 90	10×10=100	11×10=110	12×10=120
9×11= 99	10×11=110	11×11=121	12×11=132
9×12=108	10×12=120	11×12=132	12×12=144

called the *factors* of that product. Thus, as $2 \times 3 \times 4 = 24$, the numbers 2, 3, and 4 are factors of 24.

Multiplication Table

40. The product of any two numbers, neither of which is greater than 12, may be found directly from Table 4. This table should be memorized. This is very important, because success in multiplication depends largely on your ability to

name the product of any two numbers quickly. Study one section of the table at a time until you have memorized it, and then do the same for the next section. Review the sections again and again.

Do not overlook or forget the caution at the top of Table 4 that the product of zero and any number is zero. Beginners sometimes forget this fact. Remember, too, that the product of any number multiplied by 1 is equal to the number itself. This fact is demonstrated in the first multiplication table.

Practice Problems

Without referring to Table 4, multiply the following pairs of numbers and write the products on a piece of paper. Then compare your answers with those given at the back of this text. If you have made any mistakes, study the multiplication tables until you are able to get all the correct answers in 60 seconds. When you have mastered this exercise, you should have little difficulty multiplying larger numbers.

3	2	4	2	7	6	1	5	3	7
7	9	6	2	5	3	8	5	4	8
5	9	1	7	9	1	0	5	2	5
0	8	9	6	3	1	3	8	7	4
3	7	4	4	5	7	9	1	3	6
6	1	2	7	3	7	7	3	9	6
3	1	3	9	5	8	8	9	8	6
2	6	3	4	7	6	2	5	8	8
8	3	4	5	4	6	9	7	8	7
5	8	1	6	8	9	2	4	3	9

Preparing Numbers for Multiplication

41. To prepare numbers for multiplication, the multiplier is written under the multiplicand, with the units digits in the same column. It makes no difference which of the two numbers is considered to be the multiplicand because the product will be the same in either case. For example, $7 \times 8 = 56$ and $8 \times 7 = 56$. Therefore, if you want to find the product of two numbers, say 375 and 32, you may write the numbers in either of these ways:

375	32
32	375

It is customary, however, to write the larger number above the smaller one, as shown at the left, since this arrangement requires less work than the other. The product is written below the line.

Multiplying by One Digit

42. The simplest problem in multiplication is that in which the multiplier consists of only one digit, as illustrated by the following example.

EXAMPLE. Multiply 219 by 4.

multiplicand	219
multiplier	4
product	876

SOLUTION. As shown at the right, the multiplier 4 is written under the units figure 9 of the multiplicand. You begin to multiply with the units column, so the first step is to find 4×9. The fourth section of Table 4 shows that $4 \times 9 = 36$. The 6 is written below the line, directly under the units column. The remaining 30 units, or 3 tens, is carried and added to the product of 4×1. This is the second step. The product of 4 and 1 is 4, and $4 + 3 = 7$. So 7 is written below the line in the tens column. The third step is to multiply 2 by 4. The product is 8, which is written below the line in the hundreds column. Thus the product of 219 and 4 is 876. Ans.

Multiplying by More Than One Digit

43. If the multiplier consists of more than one digit, the multiplication is simply an extension of the process described in the preceding example.

EXAMPLE. What is the product of 475 and 234?

multiplicand	475
multiplier	234
first partial product	1900
second partial product	1425
third partial product	950
product	111150

SOLUTION. The numbers are written down, as shown, with their units places in the same column. Multiplication is begun by using 4 as the multiplier. The product of 4 times 5 is 20; so the 0 is written in the units column below the line and under the multiplier 4, and the 2 is carried over and added to the product of the next multiplication, which is 4×7. Thus, 4×7, plus 2, is $28 + 2 = 30$. The 0 is written under the tens column and the 3 is carried over and added to the product of 4 and 4. Now, $4 \times 4 = 16$, and adding 3 gives 19, which is written below the line, with the 9 under the hundreds column. This completes the first step, which is the multiplying of 475 by 4, giving 1900 as the first partial product.

The next step is to multiply 475 by 3, the next digit of the multiplier, thus: $3 \times 5 = 15$; write 5 directly under the multiplier 3 and carry the 1 to be added to the product of 3 and 7; this gives $21 + 1 = 22$, and the 2 is written next to the 5, the other 2 being carried and added to the product of 3 and 4; this is 3×4 plus 2, or $12 + 2 = 14$, which is written next to the 25 in the second partial product.

The third step is to multiply 475 by 2, the third digit of the multiplier. The third partial product thus obtained is 950, which is written with the 0 directly below the 2 in the multiplier. Now the three partial products in the positions shown are added, the sum being 111,150, which is the required product of 475 and 234. Ans.

Zeros in Multipliers

44. You have already learned the product of zero and any other number is zero; therefore, if the multiplier contains one or more zeros, the products of these zeros with the multiplicand will also be zeros. The following multiplications illustrate the point:

(a)

$$\begin{array}{r} 0 \\ 0 \\ \hline 0 \end{array}$$ Ans.

(b)

$$\begin{array}{r} 2 \\ 0 \\ \hline 0 \end{array}$$ Ans.

(c)

$$\begin{array}{r} 15 \\ 0 \\ \hline 00 \end{array}$$ Ans.

(d)

$$\begin{array}{r} 708 \\ 0 \\ \hline 000 \end{array}$$ Ans.

(e)

$$\begin{array}{r} 3114 \\ 203 \\ \hline 9342 \\ 0000 \\ 6228 \\ \hline 632142 \end{array}$$ Ans.

(f)

$$\begin{array}{r} 4008 \\ 305 \\ \hline 20040 \\ 0000 \\ 12024 \\ \hline 1222440 \end{array}$$ Ans.

(g)

$$\begin{array}{r} 31264 \\ 1002 \\ \hline 62528 \\ 00000 \\ 00000 \\ 31264 \\ \hline 31326528 \end{array}$$ Ans.

No matter how large or how small a number may be, multiplying it by zero gives zero as a product. This is shown in the examples from *(a)* to *(d)*. At *(e)* is shown a multiplication in which the multiplier contains one zero. The product of 3114 and the 0 is 0, and so the second partial product is 0000. The other steps in the multiplication are carried out in the usual way. At *(f)*, both the multiplicand and the multiplier contain zeros. The second partial product is 0000. At *(g)*, the multiplier contains two zeros; hence the second and third partial products are rows of zeros, as shown.

As the zeros forming partial products at *(e)*, *(f)*, and *(g)* have no value, when addition is performed to obtain the final products, these partial products may be omitted, thus shortening the work. Ordinarily, therefore, these three multiplications would be written as follows:

$$\begin{array}{r} 3114 \\ 203 \\ \hline 9342 \\ 62280 \\ \hline 632,142 \end{array} \qquad \begin{array}{r} 4008 \\ 305 \\ \hline 20040 \\ 120240 \\ \hline 1,222,440 \end{array} \qquad \begin{array}{r} 31264 \\ 1002 \\ \hline 62528 \\ 3126400 \\ \hline 31,326,528 \end{array}$$

When zeros are thus omitted, you must be careful to place the next partial product in the correct position. Just remember that the right-hand digit of a partial product is always written below the digit in the multiplier which gives that product.

Check Your Learning

82. A shortened process of repeated addition is called _____. (multiplication)

83. In multiplication, the number that is to be multiplied is called the _____. (multiplicand)

84. The number by which we multiply is called the _____. (multiplier)

85. The _____ sign is used to indicate multiplication. (times)

86. The sign that indicates multiplication is read as times, or _____. (multiplied by)

87. The result obtained when you multiply two numbers is called the _____. (product)

88. When you multiply two or more numbers together to get a product, these numbers are called the _____ of the product. (factors)

89. The product of any number multiplied by 1 is equal to the _____ itself. (number)

90. When you prepare numbers to be multiplied, the _____ is written under the multiplicand. (multiplier)

91. The product of zero and another number is _____. (zero)

Practice Problems

1. Find the product of the following:
   a) 61,483 × 6
   b) 12,375 × 5
   c) 4,836 × 47
   d) 3,257 × 246
   e) 2,875 × 302
   f) 17,819 × 1,004
2. A certain machine is capable of turning out 48 finished pieces of work in a day. At this rate, find the output of the machine in one year of 296 working days.
3. What is the total weight of 649 pumps if each pump weighs 37 lb?
4. A ranchman owns a herd of 945 cattle valued at $117 a head. How much is the entire herd worth?
5. A certain mill contains 1,830 looms, each of which produces 175 yd (yards) of cloth per week. What is the total production per week?
6. A machine turns out 50 finished pieces of work in a day. What would be the output of the machine in one year of 306 working days?
7. In a trip lasting 3 hr, how many miles can be flown by an airplane that averages 142 mph (miles per hour)?

8. If 144 copper tubes are to be bought, and the cost of each tube is $9, what will be the total cost?

9. If 40,635,000 acres of wheat produce on an average, 13 bu (bushels) to the acre, what is the total yield?

10. A power station has on hand 38 bbl (barrels) of lubricating oil. If each barrel holds 50 gal, what is the total quantity on hand?

Division

Why Division?

45. *Division* is the process of finding how many times one number is contained in another; or, to look at it another way, it is the process of separating a number into equal parts. For instance, suppose that a brass rod 32 in. long is to be divided into short pieces, each 2 in. in length. Here you would have to find how many times one number, in this instance 2, is contained in another number, 32. Or suppose that $150 is to be divided equally among 5 men. How much will each man receive? Here you must separate a number, 150, into 5 equal parts. Why 5 equal parts? Because there are 5 men!

Definitions

46. The *dividend* is the number to be divided, or to be separated into equal parts; the *divisor* is the number by which the dividend is divided; and the *quotient*, or answer, is the number which shows how many times the dividend contains the divisor.

The sign of division is ÷, which is read, divided by. Thus 54 ÷ 9 means that 54 is to be divided by 9. Notice that you must put the divisor *after* the sign of division.

Division separates a number into equal parts, or factors. It is the reverse of multiplication, which combines equal parts into a complete whole.

You should study Table 4 again until you can immediately name the factor to be found when any product is given and one of its factors is named. A good plan is to cover the second column of factors with a strip of paper or cardboard and recall them from memory. When you do this, don't take the dividends in order, select them at random. If you don't select the dividends like this, you won't be testing your ability in division.

Preparation for Divison

47. To prepare numbers for division, write the divisor at the left of the dividend, with a short curved line between them, like this: 5)225. Above the dividend and extending over it, draw a straight horizontal line from the top of the short curve, like this: $5\overline{)225}$. The numbers are then in position for the division operation. The quotient is written above the straight line, directly over the dividend.

Divisor with One Digit

48. The simplest form of division is that in which the divisor consists of only one digit. Suppose that 952 is to be divided by 7. Put down the numbers as shown at the right, following the directions given in the preceding article.

Begin the division by finding how many times the divisor 7 is contained in the first one or two digits of the dividend. Since the first digit 9 of the dividend is large enough to contain 7 once, 1 is the first digit of the quotient. It is written above the line, directly over the 9. Now 7 is multiplied by 1, and the product, 7, is written beneath the 9 and is subtracted from it, leaving 2. Bring down the next number of the dividend, which is 5, and write it after the 2, making 25, which we call the first partial dividend.

	136	quotient
divisor	7)952	dividend
	7	
first partial dividend	25	
	21	
second partial dividend	42	
	42	
	0	

Next, find the greatest number of times 7 is contained in 25. As $7 \times 3 = 21$ (which is less than 25) and $7 \times 4 = 28$ (which is more than 25), it is clear that the next digit of the quotient must be 3. So 3 is written after the 1 as the second digit of the quotient, and the 7 is multiplied by it. The product is 21, which is written beneath the partial dividend 25 and subtracted from it, leaving 4. The last

digit 2 of the dividend is now brought down and written after the 4, making 42 the second partial dividend.

The next step is to find how many times 7 is contained in 42. This is obviously 6, since $7 \times 6 = 42$. So 6 is written as the third digit of the quotient and 7 is multiplied by it. This product 42 is written beneath the second partial dividend 42 and is subtracted from it, leaving 0. This completes the division, as all the digits of the dividend have been used.

Short Division

49. In short division, only the divisor, dividend, and quotient are written. The operations are carried out mentally. Short division should always be used when the divisor consists of a single figure. The method of performing short division is illustrated here by a few examples. As you can see from the examples given, the problem solved in the preceding article could actually have been solved by short division because the divisor is a single digit.

EXAMPLE 1. Divide 4684 by 2.

SOLUTION. The numbers are written as shown at the right. The first step is to find how often the divisor, 2, is contained in the first digit, 4, of the dividend. This is obviously 2; so 2 is written directly above 4 as the first digit of the quotient. Next, the divisor 2 is divided into 6, the second digit of the dividend, the result being 3, which is written as the second digit of the quotient. Then the divisor 2 is divided into 8, the third digit of the dividend, the result being 4, which is written as the third digit of the quotient. Finally, the divisor 2 is contained 2 times in the last digit, 4, of the dividend, so that 2 becomes the final digit of the quotient, 2,342.

	2342	quotient
divisor	2)4684	dividend

EXAMPLE 2. Divide 6912 by 3.

SOLUTION. Arrange the divisor and the dividend as shown at the right. The divisor, 3, is contained 2 times in 6, the first digit of the dividend; so 2 is written as the first digit of the quotient. Next, 3 is contained 3 times in 9, the second digit of the dividend, and so 3 is the second digit of the quotient. The third digit of the dividend is 1, and the divisor 3 is larger than 1, so it cannot be contained even once in 1. Actually, 3 is contained in 1 zero times; therefore, a 0 is written as the third digit of the quotient. Now the last digit, 2, of the dividend is annexed to the 1, making 12 as a partial dividend, and 3 is contained 4 times in 12; so 4 becomes the last digit of the quotient, 2,304.

	2304	quotient
divisor	3)6912	dividend

EXAMPLE 3. What is the result of 16,784 ÷ 4?

SOLUTION. Write the numbers in the usual way, as shown at the right. The divisor, 4, is not contained even once in the first digit, 1, of the dividend. So the next digit, 6, of the dividend is annexed to the 1,

	4196	quotient
divisor	4)16784	dividend
	32	

making 16 as a partial dividend. The divisor 4 is contained 4 times in 16; therefore, 4 is written as the first digit of the quotient. Next, 4 is contained 1 time in 7, and so 1 is the second digit of the quotient. However, there is a difference of 3 after 4 is taken from 7. This 3, which is indicated by a small digit below and to the left of 8, is annexed to 8, making 38 as a partial dividend. *It should be understood that this small 3 is not written in performing short division, as the operation is done mentally. It is used here merely to make the process clear.*

The divisor 4 is contained 9 times in the partial dividend 38, and so 9 becomes the next digit of the quotient. The product of 9 and 4, or 36, is subtracted from 38, leaving 2. This 2, indicated by a small digit below and to the left of 4, has the 4 annexed to it, making 24 as a partial dividend. The divisor 4 is contained 6 times in 24, and so 6 is the final digit of the quotient, 4,196.

Check Your Learning

92. The process of finding how many times one number is contained in another is called _____.

(division)

93. In division, the number to be divided, or separated into equal factors, is called the _____.

(dividend)

94. The number by which you divide the dividend is called the _____.

(divisor)

95. The number which shows how many times the dividend contains the divisor is the _____.

(quotient)

96. You read the sign of division (÷) as _____ _____.

(divided by)

97. The process which is the reverse of division is called _____.

(multiplication)

98. In _____ division, only the divisor, dividend, and quotient are written, because the operations are carried out mentally.

(short)

Long Division

50. When the divisor has two or more digits, long division is used. This is the method that was used earlier in Art. 48. Let's work another example. Suppose that 289,456 is divided by 458. First of all, we write the numbers as shown in the solution here.

```
                    632   quotient
divisor  458)289456   dividend
           2748
           1465
first partial dividend
           1374
            916
second partial dividend
            916
```

The first step is to find how many times the divisor, 458, is contained in the first digit or digits of the dividend. It's obvious that 458 can't be contained in 2, or in 28, or in 289, because each of these is smaller than 458 itself. But 458 is contained in 2894. The largest number of times 458 is contained in 2894 is 6. Therefore, 6 is the first digit of the quotient and is written directly over the 4 in the dividend. Then we multiply 458 by 6 and write the product, 2748, under 2894. Subtracting 2748 from 2894 leaves 146. The next digit of the dividend, 5, is brought down and written after 146, making 1465, the first partial dividend.

The greatest number of times 458 is contained in 1465 is 3, so 3 is written as the next digit of the quotient. Then 458 is multiplied by 3, and the product, 1374, is written under 1465 and subtracted from it, leaving 91. The last digit, 6, of the dividend is brought down and annexed to the 91, making 916, the second partial dividend.

The divisor 458 is contained in 916 twice, and so 2 is written in the quotient as the third digit. The product of 458 and 2, which is 916, is written under 916 and is subtracted from it. This leaves 0 as a remainder. Because all digits of the dividend have been used, the division is ended. The quotient 632 is the answer.

EXAMPLE. Find the result of 106,030 ÷ 46.

SOLUTION. Write the numbers in the usual way, as shown.

```
                   2305   quotient
divisor  46)106030   dividend
             92
            140
first partial dividend
            138
              230
second partial dividend
              230
```

The divisor 46 is not contained in 1 or 10, but it is contained 2 times in 106. So 2 becomes the first digit of the quotient. Be sure to write the 2 over the 6 in the dividend. Then multiply 46 by 2 and write the product, 92, beneath 106. Subtract 92 from 106, leaving 14. The next digit, 0, of the dividend is brought down and annexed to 14, making 140. This is the first partial dividend. The divisor, 46, is contained 3 times in 140; so 3 is written as the second digit of the quotient. The product of 3 and 46, or 138, is written beneath 140 and is subtracted from it, leaving 2. The next digit of the dividend, 3, is brought down and is annexed to 2, making 23. The divisor, 46, is not contained in 23 even once, and so we say that it is contained in 23 zero times; therefore,

a 0 is the next digit of the quotient. We do not multiply this digit of the quotient by the divisor, nor do we subtract the product from the partial dividend 23, because 0 × 23 is 0 and 0 from 23 would leave 23. However, we do proceed exactly as we have done before; that is, we bring down 0, the last digit of the dividend, and annex it to 23, giving 230 as the second partial dividend. Now we see that 46 is contained in 230 exactly 5 times. Therefore, we write 5 as the last digit of the quotient, and the product of 5 and 46, or 230, under the partial dividend. When 230 is subtracted from 230, the remainder is 0 and the division is completed.

Check Your Learning

99. When the divisor has two or more digits, the process used is called _____ division.

(long)

100. In the first step of a division problem, you must find how many times the divisor is contained in the first numbers of the _____.

(dividend)

101. When the sign of division (÷) is used, the divisor is written (before, after) it.

(after)

Practice Problems

1. Divide the following:
   a) 126,498 by 58
   b) 3,207,594 by 767
   c) 11,408,202 by 234
   d) 2,100,315 by 581
2. A lot of castings weigh 11,060 lb. If the castings are alike, and one weighs 28 lb, how many are there in the lot?
3. If the driving shaft of a machine makes 9,730 turns in 35 min, how often does it turn in 1 min?
4. If sound, under certain conditions, travels at the rate of 1,118 ft per sec (feet per second), how long will it take for it to travel 23,478 ft?
5. A coil of wire 6,120 ft long is cut into 85 pieces of equal length. How long is each piece?
6. There are 5,280 ft in a mile. How many rails would it take to lay a double row of rails, each row 1 mile long, if the length of a rail is 30 ft?
7. How many electric lamps at 30¢ (cents) each can be purchased for 90¢?
8. If 16 carloads of coal weigh 46,336 lb, what is the weight per carload, assuming that each car contains the same quantity?
9. A truck uses 82 gal of gas in going 902 miles. What is its mileage per gallon?
10. If 30 water tanks of equal size contain 25,500 gal, when all the tanks are filled, how many gallons does each tank contain?

The Importance of Reviewing

51. Now that you've completed this first arithmetic text, you should have a better understanding of whole numbers, and of the basic arithmetic operations as they apply to whole numbers. And you should be able to see more clearly how important basic arithmetic really is.

If you studied this text carefully and worked out all the practice problems — making sure, of course, that you obtained the correct answers — your time has been well spent. But now you should take a little more time to review any section of this text which is not as yet perfectly clear to you. Each text in this series is a continuation of the preceding one. And so you must, for example, understand the principles of whole numbers before you go on to operations involving fractions and decimals.

When you work out the examination questions, be sure that you know just what each question asks before you try to answer it. Take your time, and work carefully. Good luck!

How to Prepare Your Work

52. One of the most important things that we can teach you is how to prepare your work so that it can be easily followed and understood by other people. We hope that the suggestions we give you here will help you avoid the shortcomings that are fairly common in a beginner's paper.

You'll find that ordinarily we correct this kind of error the first time it occurs on an examination; so be sure to examine your paper carefully for any similar errors, and to correct them yourself. This will impress the correct form on your mind. Now here are our suggestions; make the most of them.

1. Cultivate the habits of neatness and accuracy, because without these habits mistakes are bound to occur.
2. Your answers must be legible. If your handwriting is naturally poor, try to improve it by practice and carefulness.
3. Arrange your solutions in numerical order, taking care that the numbers of your answers correspond to the numbers of the questions.
4. Don't crowd your work. Don't let your work on one question run into the solution of another.

5. Be sure to state the unit in which the answer is expressed. It is just as important to know whether the correct answer is in feet, quarts, tons, or some other unit, as it is to know the number of units.

   If you like, you may treat all numbers as abstract while you perform the necessary operations and then add the correct unit to the final answer.
6. Always write the abbreviation *Ans.* after each required answer.
7. Remember that the expressions on both sides of an equality sign must be equal.
8. The sign of division (÷) means "divided by"; therefore, you should always put the *divisor* after it.
9. The minus sign (−) means "decreased by," and so the *subtrahend* must follow it.
10. Be careful with your spelling. Before you learn the definition of a new word, learn how to spell it. All misspelled words will be corrected on the first page of your examination, and any misspelled technical words will be corrected whenever they appear. This is to help you learn the correct spelling of these words. A good dictionary will be helpful if you have difficulty with spelling.

These suggestions have been put into practice in the sample examination paper reproduced here. Notice in particular the positions of the name and address and the class letters and number. Remember that items (1), (2), and (3) on your examination paper must be filled out on the first sheet of your examination, and that items (1) and (3) must be completed on each of the following sheets.

In the upper right-hand corner of the examination paper, you'll find a space designated "Study Hours." In this space please write the total number of hours that you spent on the text — that is, the number of hours you spent studying the text material, working the practice problems, and completing the programmed statements, plus the amount of time you spent preparing the examination. Suppose, for example, that you spent 13 hours studying the text and working out the practice problems and programmed statements, and 2 hours preparing the final examination. You would write "15 hours" in the space provided.

Be sure to arrange your work in numerical order, and to leave sufficient space between your answers so that your work can be easily understood and your instructor can make any necessary corrections or instruction.

Suppose that the first three questions on your final examination were as follows:

1. Find the sum of each of the following groups of numbers.
   a) 3; 6; 5
   b) 7; 10; 36
   c) 236; 141; 324

2. A salesman has traveled a 326-mile route a total of 205 times. What is the total number of miles that he has traveled?

3. Subtract each of the following pairs of numbers.
   a) 265 and 104
   b) 9,832 and 421
   c) 6,842 and 2,161

These three questions have been answered on the sample examination. Your work on the questions in Group B of the examination should be arranged in the same manner.

IDPE

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INTERNATIONAL CORRESPONDENCE SCHOOLS
Scranton 15, Pennsylvania, U. S. A.

Study Hours 15 hours

DO NOT WRITE IN THIS BLOCK
Grade____ Graded by____

Fill in (1) and (3) on ALL sheets

(1) Sheet No. 1

Answers to Examination Questions

PRINT YOUR NAME AND CORRECT ADDRESS

(3) Class Letters and Number
INX 4513281

IMPORTANT—Never omit the above, shown as the letters and numbers on the first line of your printed address which appears on all mail sent to you.

(2) Name Michael P. Jones

No. and St. / Box No. / R. D. No. 4321 N. Boulevard Drive

Post Office / State or Country Washington, D.C.

Text Title: Practical Arithmetic

No. 6750 A Pt. 1 Ed. 1

1.-

(a)	(b)	(c)
3	7	236
6	10	141
5	36	324
14 Ans.	53 Ans.	701 Ans.

2.-

326
205
1630
6520
66,830 miles) Ans.

3.-

(a)	(b)	(c)
265	9832	6842
104	421	2161
161 Ans.	9411 Ans.	4681 Ans.

Answers to Practice Problems

Article 16

1. *a)* 31,072 – thirty-one thousand, seventy-two
   b) 317,020 – three hundred seventeen thousand, twenty
   c) 1,007 – one thousand, seven
   d) 6,051 – six thousand, fifty-one
   e) 28,970,093 – twenty-eight million, nine hundred seventy thousand, ninety-three

2. *a)* six thousand, two hundred thirty
   b) twenty thousand, eight
   c) five hundred sixty-two thousand, three hundred forty-one
   d) nine thousand, seventy-five
   e) one hundred eight thousand, six hundred fourteen
   f) four hundred forty-four thousand, four hundred forty-four
   g) fourteen thousand, three hundred seven
   h) one hundred eighty thousand, nine hundred
   i) three million, six hundred thousand

3. *a)* 4,616 *c)* 11,047 *e)* 300,000 *g)* 1,776
   b) 5,280 *d)* 30,903 *f)* 534,825 *h)* 13,800,200

4. *a)* 4,999, 5,000, 5,001, 5,002
   b) 52,630, 52,631, 52,632
   c) 660,000, 660,001

Article 24

10	11	10	4	12	9	9	10	7	15
5	17	10	13	12	2	3	13	9	9
9	8	6	11	8	14	16	4	12	12
5	7	6	13	12	14	10	14	16	14
13	11	5	11	12	15	11	11	11	16

Article 27

1. 1,134
2. 21,676
3. 43,167
4. 559,153
5. 636,565
6. 125,431,367

Article 28

1. 94
2. 75
3. 264
4. 297

Article 30

1. 5,105 lb
2. 58,688 parts
3. 1,875 ft
4. 4,029 miles
5. 184,438 people
6. 945,267 vehicles
7. 13,220 gal
8. $8,535
9. *a)* 144,895
   239,360
   155,383
   b) 27,385
   82,447
   95,266
   97,875
   156,052
   80,613
   c) 539,638

Article 34

1.

(a)	*(b)*	*(c)*	*(d)*	*(e)*	*(f)*	*(g)*	*(h)*	*(i)*	*(j)*
1	2	6	2	6	0	2	1	1	4
(k)	*(l)*	*(m)*	*(n)*	*(o)*	*(p)*	*(q)*	*(r)*	*(s)*	*(t)*
4	8	5	9	8	3	8	8	5	9

2. *a)* 1 *c)* 3 *e)* 6 *g)* 0
   b) 9 *d)* 3 *f)* 5 *h)* 5
3. *a)* 6 *c)* 9 *e)* 5 *g)* 8
   b) 9 *d)* 8 *f)* 8 *h)* 6
4. $4
5. 6 cows

Article 36

1. *a)* 253 *b)* 622 *c)* 31,719 *d)* 47,890
2. *a)* 78 *b)* 3,593 *c)* 10,396 *d)* 282
3. *a)* 7,888 *b)* 3,918

Article 37

1. 35 points
2. $3,069
3. 4,500 gal
4. 88,240 lb
5. $2,655
6. 93,760,000 miles

Article 40

21	18	24	4	35	18	8	25	12	56
0	72	9	42	27	1	0	40	14	20
18	7	8	28	15	49	63	3	27	36
6	6	9	36	35	48	16	45	64	48
40	24	4	30	32	54	18	28	24	63

Article 44

1. *a)* 368,898
   b) 61,875
   c) 227,292
   d) 801,222
   e) 868,250
   f) 17,890,276
2. 14,208 pieces
3. 24,013 lb
4. $110,565
5. 320,250 yd
6. 15,300 pieces
7. 426 miles
8. $1,296
9. 528,255,000 bushels
10. 1,900 gal

Article 50

1. *a)* 2,181
   b) 4,182
   c) 48,753
   d) 3,615
2. 395 castings
3. 278 times
4. 21 sec
5. 72 ft
6. 352 rails
7. 3 lamps
8. 2,896 lb
9. 11 miles per gallon
10. 850 gal

Practical Arithmetic

Serial 6750A PART 1 Edition 1

Examination Questions

Notice to Students.—Study this instruction text thoroughly before you try to answer the following questions. Read each question carefully and be sure you understand it. Then write the best answer that you can. If your answer requires a mathematical solution, show enough of your work to indicate the method you used. Unless you do this we will have no way of knowing how you obtained your answer and your work will not be considered complete. After you have answered all the questions, double-check them to make sure that this is the best work you can do. Don't omit any questions.

Mail your work to us as soon as it is finished. DON'T HOLD IT until another examination is ready.

GROUP A

Answer each of the following statements by giving the number of the statement and the word or words that you think will correctly complete the statement. Each correct answer in Group A is worth one point.

1. A whole number is often referred to as a(n) INTEGER.
2. Addition is the process of finding the SUM of two or more numbers.
3. The difference between two numbers is found by SUBTRACTION.
4. The equals sign (=) indicates a statement of EQUALITY.
5. A number which is not an abstract number must be a(n) CONCRETE number.
6. A number used to indicate the position of an object in a series is called a(n) CONCRETE number.
7. When a number is divided by another number, the answer obtained is called the QUOTIENT.
8. The ordinal number corresponding to 32 is written in words as THIRTY SECOND.
9. When two or more numbers are multiplied together, the answer is called the PRODUCT.
10. When any number is multiplied by zero, the result is ZERO.

GROUP B

When answering the questions in this section, be sure to show all of your calculations. If the complete solution for each problem is not shown, you will not receive full credit for the answer. The point value of each question is given in parentheses after the number of the question.

11. (8) *a)* Find the sum of each of the following.

6	8	10	22
3	5	12	41
2	7	56	13

b) Find the difference between the following.

6	12	85	608
3	6	27	241

12. (9) Add each of the following groups of numbers.
a) 21; 35; 86; 10
b) 727; 2,324; 6,513
c) 27,631; 5,917; 14,826

13. (9) Subtract each of the following numbers.
a) 789 and 213
b) 68,918 and 26,362
c) 2,231 and 279

14. (9) Multiply the following.
a) 32,764 and 23
b) 6,235 and 467
c) 5,218 and 3,056

15. (9) Find the quotient.
a) 82,688 ÷ 38
b) 69,795 ÷ 235
c) 1,451,736 ÷ 572

16. (6) *a)* Find the sum of 386 and 402.
b) Find the product of 386 and 402.

17. (10) A company has a profit of $54,828, which is to be shared equally by 1,523 stockholders. How many dollars will each stockholder receive?

18. (10) Find the product of each of the following groups of numbers.
a) 27 × 63 × 24
b) 239 × 486 × 25 × 3

19. (10) A man's estate valued at $68,925 was divided as follows: $30,000 to his wife, $1,500 to charity, $7,425 in payment of debts, and the amount remaining to his three (3) children in equal shares. How much did each child receive?

20. (10) If the total cost of three (3) motors is $540, what will be the total cost of sixteen (16) of these motors?